An $\mathcal{H}$-Matrix Accelerated Direct Solver for Fast Analysis of Scattering from Structures in Layered Media

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Frequency-domain mixed-potential integral-equations are commonly used for analyzing scattering from structures residing in layered backgrounds (K. A. Michalski and D. Zheng, IEEE TAP, 38(3), 335–344, 1990). Their traditional direct method-of-moments solution based on LU-decomposition is limited to small problems, however, because it requires $O(N^2)$ operations to fill the $N \times N$ dense impedance matrix (with a large constant in front of the complexity estimate because of the high cost of computing Sommerfeld integrals), $O(N^3)$ operations to factorize the impedance matrix, and $O(N_{\mathrm{rhs}} N^2)$ operations to find the solution for N_{rhs} different excitations. Moreover, modern fast iterative algorithms that enabled the solution of extreme problems for homogeneous backgrounds are often inapplicable, inefficient, or must be modified significantly (K. Yang and A. E. Yılmaz, CEM Int. Workshop, 2013) for layered backgrounds because of the more complicated Green functions. In this article, a fast direct solver based on the hierarchical matrix ($\mathcal{H}$-matrix) framework (W. Hackbusch, Computing, 62, 89–108, 1999) is used to accelerate the analysis of scattering from structures residing in layered media.

In the $\mathcal{H}$-matrix approach, first the structure of interest is partitioned into subdomains such that the impedance matrix is divided into a hierarchy of blocks representing interactions between pairs of subdomains. Then, matrix blocks that correspond to the interactions between well-separated subdomains are represented via low-rank factorizations, which results in reduced-complexity matrix operations and compressed storage. The savings that can be achieved are dependent on the partitioning scheme and the low-rank factorization strategy. Even though the $\mathcal{H}$-matrix framework has been used effectively to accelerate the analysis of scattering from structures residing in homogeneous backgrounds in the low, intermediate, and, in some cases, even in the high-frequency regime (W. Chai and D. Jiao, IEEE TAP, 57(10), 3147–3159, 2009), the method's performance is different for layered medium problems. Indeed, the compression rate of the impedance matrix blocks can be quite different for layered backgrounds because of the more complex interaction mechanisms and more complicated dependence of the rank on distance (Y. Brick, V. Subramanian, and A. E. Yılmaz, URSI Rad. Sci. Meet., 2015). At the conference, the performance of the algorithm with various partitioning schemes and low-rank factorization strategies will be contrasted to that of an FFT-accelerated iterative algorithm (K. Yang and A. E. Yılmaz, CEM Int. Workshop, 2013) for representative scattering problems in layered media. Specifically, the methods will be used to accelerate surface- and volume-electric-field integral equations pertinent to the analysis of scattering from buried objects in the UHF band, *S*-parameters of a package-level interconnect network, and controlled source electromagnetic surveys.