Rank Deficiency of Impedance Matrix Blocks for Layered Media

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The development of fast and robust layered-medium integral-equation solvers continues to lag behind that of their free-space counterparts. For example, in the last decade, accelerated direct solution algorithms (P. G. Martinsson and V. Rokhlin, *J. Comput. Phys.*, 205, 1-25, 2005) have flourished almost exclusively for free-space integral equations. Such algorithms generally involve the computation of a low-rank matrix representation of sub-matrices (blocks) of the method-of-moments impedance matrix $\mathbf{Z}$. Typically, an (often accelerated) algebraic rank-revealing procedure is used to analyze the blocks of $\mathbf{Z}$, which represent interactions between different source and observer subdomains of the structure of interest, and to compute their low-rank representations. This representation of $\mathbf{Z}$ via low-rank blocks enables the rapid computation of its compressed inverse form, which can in turn be rapidly applied to multiple right-hand-side vectors that correspond to different excitations. The expected ranks of the blocks must be taken into account when designing these algorithms, specifically when selecting (i) a favorable strategy for partitioning $\mathbf{Z}$ into blocks (structure into subdomains) and (ii) an efficient rank revealing procedure (Y. Brick and A. Boag, *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, 58, 2405-2417, 2011). A block's rank is dictated by the form of the integral kernel, the size of the corresponding source and observer subdomains, and, for some solution techniques, the distance between them. For structures residing in homogenous backgrounds, due to the form of the free-space Green's function, the rank can be estimated well given the sizes and separation of the source and observer subdomains. For structures residing in planar layered media, however, the rank is more difficult to estimate because it can also depend on the thickness and material properties of the layers and the subdomains' positions relative to the interfaces; i.e., typical interaction and loss mechanisms in layered media, such as reflection, refraction, surface waves, and guided modes, can affect the rank. Such effects can become particularly pronounced for media with significant contrast in the layers' permittivity and/or conductivity and should be properly accounted for in the design of an accelerated direct solver.

To support the development of accelerated direct layered-medium integral-equation solvers, this article presents a numerical study on the rank's behavior in layered media. To this end, a rank-revealing analysis of interactions is performed for various setups. To be able to reveal the rank for large blocks, both inter- and intra-layer interactions are represented by using sampling grids that are tailored to the medium. These simulations quantify the rank's dependence on the subdomains' size and separation (transverse to and along the stratification direction) for various backgrounds. At the conference, we will discuss the implications of the results on the design and expected performance of different classes of direct solvers.