Discontinuous Galerkin Boundary Element Methods for Maxwell's Equations: Discretization, Formulation and Preconditioning

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Integral equation methods are often solved via the Galerkin method, which is based on a variational formulation in suitable trial and testing function spaces. Therefore, conforming boundary element spaces defined on the target's surface are commonly required. However, generating conformal discretizations for electromagnetic (EM) high-fidelity simulations is far from trivial, as the complexity of engineering applications increases at a fast pace. Moreover, most conformal boundary element spaces are tightly associated with the underlying discretization. Therefore, mixing different types of basis functions, employing different discretizations and/or incorporating the underlying physics to construct special basis functions within local regions pose immense challenges.

This research investigates a discontinuous Galerkin boundary element method (DG-BEM) for time-harmonic Maxwell's Equations, which employs discontinuous trial and testing functions without continuity requirements across element boundaries. Initial work has been done to demonstrate the potential of DG-BEM in solving the EM wave scattering from PEC objects in (Z. Peng, K.-H. Lim and J.-F. Lee, IEEE Trans. vol. 61, no. 7, 3617-3828, 2013). We have shown that the DG-BEM provides the same order of accuracy and convergence behavior compared to that of the traditional Galerkin method. The stability and convergence are validated for targets with sharp edges and corners. Moreover, the discrete formulation only requires the evaluation of weakly singular integrals, which can be easily performed with high order accuracy.

Along this line of research, we have made several important advancements including: (**i**) investigation of antisymmetric and symmetric DG weak forms, and an a-priori mathematical analysis to determine the asymptotic behavior of the interior penalty stabilization function. Quasi-optimal convergence measured in $\mathbf{L^2}$ norm is illustrated through numerical experiments; (**ii**) a hybrid DG boundary element approximation including both piecewise linear basis functions and plane wave basis functions. The goal of this study is to develop an accurate yet efficient approximation of surface currents for a desired level of accuracy. The oscillatory nature of solutions is incorporated into the construction of trial spaces; (**iii**) parallel, scalable domain decomposition algorithms for the efficient and robust solution of the DG-BEM matrix equations. The merits of proposed method are justified through different types of real-world applications ranging from high-definition jet aircrafts to radar stealth objects to semiconductor quantum dots and plasmonic nanoantennas.