ET-AIM Accelerated Analysis of Scattering from Inhomogeneous Objects with Time-Varying Permittivity

Guneet Kaur and Ali E. Yılmaz
Department of Electrical and Computer Engineering
University of Texas at Austin, Austin, TX 78712, USA

Scattering from inhomogeneous objects whose constitutive relationship for the electric-flux density can be expressed in terms of a time-varying permittivity $\varepsilon(\mathbf{r},t)$ as $\mathbf{D}(\mathbf{r},t)=\varepsilon(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)$ is of interest in a variety of high-power and optical applications; e.g., space-time modulation of permittivity has been recently explored as a mechanism for designing non-reciprocal ring resonators (D. L. Sounas and A. Alù, *IEEE Antennas Propagat. Soc. Int. Symp.*, 2014). Because of frequency mixing introduced by the temporal change in ε, traditional frequency-domain analysis is inapplicable and transient analysis in time-domain is the natural approach for finding the scattered fields. Indeed, various finite-difference time-domain (X. Liu and D. McNamara, *Int. J. Infrared Millim. Waves*, 28(9), 2007) and time-domain integral-equation (F. V. Fedotov, *et. al.*, *J. of Lightwave Tech.*, 21(1), 2003) methods have already been used for this analysis. Such time-domain simulations, however, can be very expensive when ε varies slowly compared to the rate of change of fields (as is often the case). This is because the time-step size used must be small enough to resolve the fast variations of the fields while the time interval that is analyzed must be long enough to capture all the effects of the varying material property (determined in part by the largest distance the fields must travel across the object and any resonances). As these two time scales become more disparate, the computational costs of time-domain methods increase and alternative methods become more attractive. For example, the number of time steps that must be simulated can be drastically reduced by envelope-tracking methods (G. Kaur and A. E. Yılmaz, *Proc. IEEE Antennas Propagat. Soc. Int. Symp.,* 2011) that use larger time steps that are dictated by the bandwidth rather than the maximum frequency content of the fields.

This article presents an envelope-tracking solution of the volume electric-field integral equation (VEFIE) pertinent to the analysis of wave propagation though spatiotemporally modulated lossy dielectrics, where the material properties vary much slower than the fields. The proposed method is based on the 2-field formulation (B. Shanker *et al.*, *Radio Sci.*, 36, RS2007, 2004); i.e., the envelopes of both the "conduction current corrected flux density" and the total electric field are discretized and solved simultaneously. An auxiliary equation relating these two envelopes is obtained by applying the product theorem of the Hilbert transform in (E. Bedrosian, *Proc. IEEE*, vol. 51, 1963), which is valid whenever the maximum rate of variation of ε is smaller than the minimum frequency component of the fields. The coefficients from the auxiliary equation are multiplied explicitly outside instead of adding their contribution inside the impedance matrices as originally presented for the 2-field formulation. This facilitates a simple framework to incorporate time-varying material properties at each time step. The computations are accelerated by using the envelope-tracking adaptive integral method (ET-AIM) (Kaur and Yılmaz, *Proc. USNC/URSI Rad. Sci. Meet.*, 2012), which is adapted to the 2-field formulation. The necessary modifications for ET-AIM include separately saving inter/anterpolation coefficients for half basis functions and an additional $l^{\max}$ number of temporal FFT computations at each spatial point at each time step to account for the terms that arise from the auxiliary equation; here, $l^{\max}$ denotes the length of the temporal basis function.