Domain Decomposition Methods with Fast Direct Solvers for Multi-scale Electromagnetic Problems

Yang Shao*[1,2], Kheng-Hwee Lim[2], Zhen Peng[1], and Jin-Fa Lee[2]

[1] Dept. of Electrical and Computer Engineering, University of New Mexico, Albuquerque, NM, 87131

[2] ElectroScience Lab, The Ohio State University, Columbus, OH, 43212

The purpose of this paper is to investigate a hybrid, fast solution strategy for large, multi-scale electromagnetic problems. Applications of scattering problem from large multi-scale objects often lead to ill-conditioned matrix. We may take an high-definition aircraft shown in Fig. 1 as an example. The ill-conditioning may contribute to the following aspects: (*i*) sharp edges and corners, (*ii*) ill-shaped triangular discretizations, (*iii*) deep cavities where the standing wave phenomena are the most prominent physical effect, and (*iv*) low frequency Instabilities.

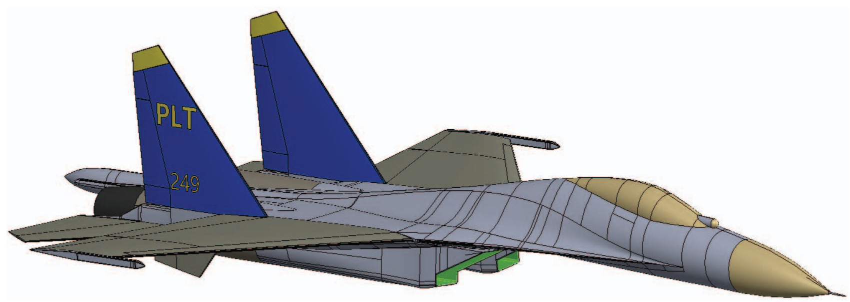

Figure 1: A high-definition jet aircraft.

In this work, we propose to study a hybrid non-conformal domain decomposition method (DDM) to tackle these difficulties. Volume-based finite element (FE) solvers and surface-based IE solvers can be applied to individual sub-domain problems based on local EM characteristics. Fast direct solver is utilized for solving local sub-domain matrices that have singularities. Schwarz preconditioning is used for the DDM system matrix. Moreover, in order to exploit the rank deficiency property exhibited in the interaction matrices between sub-systems, we will utilize the advantages of the hierarchical multi-level fast multiple method (H-MLFMM). We first employ the multi-level skeletonization to construct effective basis functions, the so-called skeletons, from Huygen's equivalent sources associated with individual sub-systems. The interactions between sub-systems will be computed using selected skeletons, and the DDM iteration will be performed on the compressed skeleton system. In this way, instead of directly applying DDM scheme to the original full-scale system with N degree of freedom (DoF), we construct a skeleton-based compressed system to reduce the DD matrix dimension from $\mathcal{O}(N)$ to $\mathcal{O}(M)$, where $M \ll N$. We anticipate this hybrid DDM will significantly improve the effective and stability of the numerical solution.