

## Domain Derivatives in Scattering from Rough Surfaces

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The inverse scattering solution of shape and/or material parameter reconstruction is often posed as a problem in nonlinear minimization of an objective function with respect to  $N$  (usually large) number of unknown model parameters characterizing the scatterer. The minimization procedures are usually iterative, and require the gradient of the objective function in the unknown model parameter vector in each stage of iteration. For large  $N$ , finite-differencing becomes numerically intensive, and an efficient alternative is domain differentiation in which the full gradient is obtained by solving a single scattering problem of an auxiliary field using the same scattering operator as that of the forward solution. One technique in this direction is the so-called *adjoint field method* which obtains the gradient by variationally minimizing an augmented objective function that includes the reduced wave equation via a Lagrange multiplier. Results are reported mostly for compact objects. This paper presents the domain derivative calculation of the gradient for a locally perturbed, infinitely long dielectric interface. The method is non-variational, and algebraic in nature in that it evaluates the gradient by directly domain differentiating the scattering equations. The computations are straight forward and easy to follow. The mathematical transformation of the scattering problem into the corresponding problem for the differentiated fields can be visualized explicitly. The formulation of and the motivation behind introducing the auxiliary field are explicitly demonstrated. Closed-form analytic expressions are obtained for the gradients for electromagnetic TE/TM scattering from dielectric rough surfaces, and for scalar wave scattering from Neumann and Dirichlet rough surfaces. Results are compared with those for compact scatterers. Finally, the relationship between our results and Lorentz reciprocity is pointed out and clarified. The analysis involved in this work also suggests a couple of other approaches to this problem. Depending on the application at hand one approach may be more convenient than the other.