# Average Transition Conditions for Electromagnetic Fields at a Perfectly Conducting Metascreen

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Much research has been carried out in recent years on the use of metasurfaces as an alternative to metamaterials for achieving unusual functionality in the transformation of electromagnetic waves. For one type of metasurface, known as a metafilm, which consists of a surface array of isolated metallic or dielectric resonant scatterers, the interaction of the macroscopic electromagnetic field with a metafilm can be modeled quite accurately by the use of generalized sheet transition conditions (GSTCs). Other types of metasurface have not been modeled in this way, however, and recourse must be had to completely numerical simulations. In this paper, we will derive analytically the transition boundary conditions that describe a *metascreen*: a surface array of electrically small apertures in a perfectly conducting screen. The procedure is analogous to that used for a metafilm. For a screen of zero thickness in the plane $z = 0$, each aperture is characterized by its polarizabilities, and the acting field at each aperture is due to external sources plus and to all other apertures. This results in a self-consistent set of equations that can be rewritten in the form of the desired boundary conditions. We find that the tangential macroscopic electric field must be continuous across the metascreen, and equal to

$$\mathbf{E}\big|_{z=0} \times \mathbf{a}_z = j\omega\mu_{\rm av}\overset{\leftrightarrow t}{\boldsymbol{\pi}}_{ms} \cdot \left[\mathbf{H}_t\right]_{z=0^-}^{0^+} + \frac{1}{\epsilon_{\rm av}}\nabla_t \left\{\pi_{es}^{zz}\left[D_z\right]_{z=0^-}^{0^+}\right\} \times \mathbf{a}_z$$

in terms of the jumps of tangential $\mathbf{H}$ and normal $\mathbf{D}$. Here $\pi_{es}^{zz}$ and $\overset{\leftrightarrow t}{\boldsymbol{\pi}}_{ms}$ are the electric and magnetic *surface porosities* of the metascreen (essentially the aperture polarizabilities per unit area). When the thickness $h$ of the metal screen is not zero, the Bethe small-aperture theory must be modified (R. L. Gluckstern and J. A. Diamond, *IEEE Trans. Micr. Theory Tech.*, 39, 274-279, 1991). For a metascreen occupying $-h/2 < z < h/2$, the equation above still holds for the *average* electric field at the metascreen, but with the jumps calculated between $z = \pm h/2$ and with different equations for the surface porosities. The tangential electric field will no longer be continuous across the metascreen, but rather has a jump expressed in terms of the surface susceptibilities and average fields at the metascreen, similar to what is found for a metafilm:

$$\left[\mathbf{E}\right]_{z=-h/2}^{h/2} \times \mathbf{a}_z = j\omega\mu\overset{\leftrightarrow t}{\boldsymbol{\chi}}_{ms} \cdot \mathbf{H}_{t,\rm av} - \frac{1}{\epsilon}\nabla_t \left(\chi_{es}^{zz} D_{z,\rm av}\right) \times \mathbf{a}_z$$

These GSTCs are used to compute plane wave reflection and transmission coefficients analytically, and these results are compared to those from a full-wave finite-element simulation. Agreement is found to be good when the lattice constant of the aperture array is sufficiently small.