Stored electromagnetic energy expressed in the fields, currents, and input impedance

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Stored electromagnetic energy is used in optimization of antennas and to derive physical bounds. The stored electromagnetic energy can be evaluated from the fields around the antenna, the currents on the antenna, and the input impedance. The different proposals are in general different even if they produce similar results for small structures.

Chu (1948) used lumped circuits models of the spherical modes to determine the stored energy and the associated Q-factor outside a spherical volume. Collin and Rothschild (1964) interpreted the stored energy as the difference between the energy density in the electromagnetic fields and the energy in the radiated field. Harrington (1972) proposed a numerical approach based in frequency differentiation of the electric field integral equation impedance matrix to estimate the energy. Geyi (2003) and Vandenbosch (2010) expressed the stored energy in the current density on the antenna structure. Brune (1931) synthesis is used by Gustafsson and Jonsson (2015) to estimate the stored energy directly from the input impedance. Differentiation of the input impedance (Yaghjian and Best 2005) is an alternative methodology to estimate the Q-factor and fractional bandwidth. This Q-factor is alternatively expressed in the current density and its frequency derivative by Capek etal 2013.

Here, we present an overview of stored electromagnetic energy and compare the resulting Q-factors for several antennas. We also present an extension to lossy and temporally dispersive media based on differentiation of the method of moment matrix (Gustafsson etal 2014). The resulting Q-factors agree with the Q-factors from Brune synthesized circuits as long as the Q-factor is large and the temporal dispersion is weak. The Q-factor from the differentiated input impedance is also generalized to temporally dispersive media and expressed in the current density.