An Approximate Characterization of the Distortion of Narrowband Signals by Linear Time-Invariant Systems

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The increased demand for data communication at high rates has made it important to transmit this information as accurately as possible, and to characterize its distortion when passed through a communication link. Ultrawideband (UWB) links have applications in radar, location, tracking and sensor applications. Although some parts of such links (amplifiers, for instance) are nonlinear and/or time-varying, much of the signal path (including the antennas) can typically be characterized as a linear, time-invariant (LTI) system. In this paper, we will review a widely-used measure of distortion called the fidelity factor, and provide a simple analytical approximation for it that can be used to quickly assess the extent to which pulsed signals are degraded when passed through an LTI system.

Let $f(t)$ be the input signal pulse to the system, $F(\omega)$ its Fourier transform, and $H(\omega)$ be the transfer function of the LTI. Then the fidelity factor (D. Lamensdorf and L. Susman, *IEEE Ant. Prop. Mag.*, 36 (1), 20-30, 1994) is defined as

$$\mathrm{FF} = \max_{\tau} \frac{\left|\mathrm{Re}\left[\int_0^\infty H(\omega)\left|F(\omega)\right|^2 e^{j\omega\tau}\, d\omega\right]\right|}{\sqrt{\int_0^\infty \left|F(\omega)\right|^2\, d\omega \int_0^\infty \left|H(\omega)F(\omega)\right|^2\, d\omega}} \tag{1}$$

If the input signal bandwidth is sufficiently narrow, we may approximate everything except $\left|F(\omega)\right|^2$ in this expression as a Taylor series about the center frequency ω_0 of the input pulse, and carry out the resulting integrals term by term up to second order, resulting in a closed-form approximation for FF. The result will be compared with a numerically exact evaluation of (1) for several test cases, including a realistic UWB antenna, and agreement shown to be very good if the bandwidth is not too large.

The approximate formula obtained here shows how various parameters of the transfer function at ω_0 (such as group delay, flatness of the group delay and amplitude flatness) affect the fidelity factor. This in turn could guide the engineer in designing a distortion compensation network to alleviate the distortion for a given combination of input pulse and linear system.