Higher Order (in Time) Stable PWTD-Accelerated Time Domain Integral Equation Solver

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Integral equation based methods for transient analysis have seen a rapid growth in the state of the art over the past decade. The fundamental bottlenecks are well understood and there are prescriptions to their resolution. The first being computational complexity. Classical methods scale as $\mathcal{O}(N_t N_s^2)$ where N_t and N_s are the number of temporal and spatial degrees of freedom. Methods to ameliorate this cost have been developed, and are the plane wave time domain method (PWTD) (A. A. Ergin, B. Shanker, and E Michielssen, Journal of Computational Physics 146 (1), 157-180, 1998) and the time domain adaptive integral method (TDAIM) (A. E. Yilmaz, J. M. Jin, and E. Michielssen, IEEE Transactions on Antennas and Propagation, 52, 2692-2708, 2004). These methods reduced the cost to $\mathcal{O}(N_t N_s \log^2 N_s)$, and $\mathcal{O}(N_t N_c \log^2 N_c)$, where N_c is the number of auxiliary in grid points on the Cartesian grid. The second bottleneck is late time stability. Papers attempting to address this issue have been ongoing since the 1960's. It is only recently that some resolution has been brought to this issue; see (A. J. Pray *et. al.*, IEEE Transactions on Antennas and Propagation, 62, 6183-6191, 2014) and reference therein. However, integration of fast methods with late time stable techniques is still a challenge.

In this work, we will work to integrate late time stable methods developed by our group with the PWTD method. While this was a work that we embarked upon last year, incorporating PWTD into the solution system brings unexpected results. The PWTD algorithm yields the derivative of the field as opposed to the field. Simple numerical integration yields results that have a slowly growing late time instability. In this paper, we seek a resolution to this problem by developing accurate integration schemes. To do so, we shall explore both Runge-Kutta (explicit and implicit) methods as well as variational based integrators. The results of these methods and their influence on stability will be presented at the conference.