On the Computational Complexity of Polynomial Chaos Expansion for FDTD-Based Uncertainty Analyses

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Limitations in measurement precision and variability in manufacturing processes leads to statistical uncertainty in the reported electromagnetic properties of materials. There is a growing interest in extending the capabilities of deterministic full-wave computational techniques such as FDTD to enable efficient analysis of electric and magnetic field uncertainties (i.e., mean and variance) due to uncertainties in the material properties. The use of large numbers of deterministic simulations to compute field variability, i.e. the Monte Carlo approach, yields accurate results but is computationally very expensive.

The polynomial chaos expansion (PCE) technique is used for uncertainty quantification of functions of multiple random variables in stochastic differential equations. It has been successfully integrated into the FDTD framework to quantify uncertainty with reduced computational cost relative to the Monte Carlo method (R. S. Edwards, A. C. Marvin and S. J. Porter, IEEE Trans. Electromagn. Compat., 52, 1, 155-163, Feb. 2010; A. C. M. Austin and C. D. Sarris, IEEE Trans. Microw. Theory Tech., 61, 12, 4293-4301, Dec. 2013). PCE-FDTD approximates fields as polynomial functions of uncertain material parameters. Specific types of conjugate polynomials are used with a Galerkin formulation to obtain FDTD update expressions for the polynomial coefficients. The computational complexity of the PCE-FDTD algorithm is proportional to $P = (N + D)!/(N!D!)$, where N is the number of random variables and D is the polynomial order. The complexity of PCE-FDTD increases rapidly with N and D.

In this paper we explore the polynomial order D required to accurately model the fields over a range of uncertain parameters for scenarios involving scatterers of varying sizes, shapes, and positions. The parameter space of this study spans a wide range of dielectric constants and loss tangents, as well as uncertainty levels. The results show that a large variation in polynomial order, from $D = 1$ to $D > 10$, is required to obtain field means and variances comparable to those obtained using Monte Carlo methods. Thus, PCE-FDTD can have extremely high computational cost in certain scenarios and be very efficient in others.