Evolution Equations for the Oscillations in a Cavity Filled with a Dynamic Medium

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Development of the time-harmonic electromagnetic theory dates back to the end of 19th century stating that all the electromagnetic field quantities vary harmonically in time. This assumption, being formalised in the system of Maxwell's equations (ME) with time derivative, changes the *hyperbolic* type of the partial differential equation into the *elliptic* type. Therefore, the time-harmonic solutions to the ME of *elliptic* type do not satisfy the causality principle and have difficulties with the relativistic transformations. Progress of the time-domain field theory has started via the development of computational electromagnetics in the second part of the 20th century. Numerical techniques are capable of providing huge volumes of numerical data; however, a deep physical insight to the data can be achieved under the presence of analytical results. Nevertheless, analytical theory of time-domain electromagnetics goes through an initial step of its development.

Our goal in this study is to present an analytical time-domain solution for the forced oscillations in a cavity filled with a *dynamic medium* in compliance with the causality principle. Maxwell's equations and the *motion equation* for the dynamic medium should be solved simultaneously under the *initial conditions*.

The study is presented as follows. First, formulation of the time-domain problem is given, where the initial conditions are specified as well. A self-adjoint operator, which acts on the spatial variables only, is separated from the Maxwell’s equations. The operator eigenvalue equation is obtained. Solving it yields definiton of the vector elements of the basis, which have physical sense of the *solenoidal* and *irrotational* cavity modes. This results in the *modal field expansions*, where the modal amplitudes are scalar functions dependent on time yet to be known.

Projecting of Maxwell's equations and the initial conditions onto the modal basis yields the system of ordinary differential equations with the time-derivative for the *modal amplitudes*. They are supplemented with the appropriate initial conditions. The differential equations with time-derivative are called as evolutionary ones in mathematics. The evolutionary equations jointly with the initials conditions originate the *Cauchy problem*. As long as the Cauchy problem plays principle role in our method, we named that as the Evolutionary Approach to Electromagnetics (EAE). Eventually, we arrive to the exact explicit solutions for the modal amplitudes. They are presented in the form of the convolution integrals where a given signal function can be a variant. No other restriction on the signal functions should be applied except their *integrability*.

This approach was recognized recently as an alternative to the classical time-harmonic field method: (O. A. Tretyakov and F. Erden, "Evolutionary Approach to Electromagnetics as an Alternative to the Time-Harmonic Field Method," *IEEE AP-S and USNC/URSI Radio Science Meeting*, 8-14 July 2012, Chicago, US). Graphical results from several implementations of the EAE for cavity problems illustrating the time dependence of the *irrotational* and *solenoidal* modes will be exhibited at the Conference.