Time-Domain Response of Parity-Time Symmetric Structures: Causality, Stability and Bandwidth Considerations

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Parity-Time (PT) symmetry has been recently introduced in quantum mechanics as a condition allowing non-Hermitian systems to support real eigenvalues, and as a result to be physically permissible. Although PT symmetry in quantum mechanics is still under dispute, it has attracted significant attention in classical electromagnetic and acoustical systems, where it is manifested by spatially balanced amounts of loss and gain, and leads to unique phenomena, such as unidirectional invisibility, laser-absorber modes, and negative refraction. To date, most of the studies on PT symmetric systems have focused on their steady-state time-harmonic response. However, since such systems are active, they are potentially unstable, and investigation of their time-domain properties is crucial in order to determine whether such systems are physically possible.

Here we study the time-domain properties of PT-symmetric structures and determine the relation between causality, stability and bandwidth. In order to better understand the relation between such measures, we focus our discussion on a few examples of interest for electromagnetic applications, such as the case of a PT-symmetric planar lens constituted by two metasurfaces, and a PT-symmetric cylindrical cloak, consisting of a PT-symmetric metasurface around a cylindrical object. The operation of such devices in steady-state has been described by the authors in recent papers, and it is based on absorption of the incident power from the passive side of the system and emission of a signal synchronized to the incident one from the active side. Since the active side is excited by waves creeping from the passive region, and because such waves propagate along a longer path than the incident wave, it is evident that these PT-symmetric systems have to comply with causality limitations. Our analysis shows that the proposed structures support resonant eigen-modes that, in a stable scenario, can progressively generate the required phase profile to achieve the desired steady-state configuration, in perfect agreement with the time-harmonic solution of Maxwell equations. These modes are stable for a minimum value of their quality factor or, equivalently, for a maximum value of the bandwidth of the structure, a condition which can be satisfied by appropriately selecting the dispersion properties of the active side. Interestingly enough, the dispersion which leads to a stable design can be totally different from the dispersion that leads to a locally-stable active metasurface, demonstrating the importance of the interaction between the different parts of the PT-symmetric system. Our findings provide fundamental insights into PT-symmetric phenomena and show the significance of time-domain analyses to unveil such mechanisms and design devices with optimum characteristics. Our work also sheds new insights into the stability of active metamaterial and antenna systems, including non-Foster elements.