# Evaluation of 4-D Reaction Integrals

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A number of researchers have focused recently on the evaluation of the 4-D, singular reaction integrals resulting from surface integral equation formulations (A.G. Polimeridis et al., IEEE Trans. Ant. Propagat., 61, 3112-3122, 2013). Recently the authors showed that for co-planar elements, these promising approaches can be generalized and improved by applying the divergence theorem twice, once each for the source and test domains, to obtain a general formula valid for self, edge-, and vertex-adjacent, as well as separated element pairs. Unlike approaches first transforming to barycentric coordinates, the result is also relatively insensitive to element shapes. Furthermore, it is found that the radial integrals can be evaluated on a triangular region in the radial source/test variable plane, where efficient Gauss quadrature rules exist. And finally, the remaining contour integrals can be transformed to cancel the variation of the static kernel of the integrand, significantly improving the scheme's convergence.

Recently, we have focused further on the evaluation of the radial integrals. For homogeneous media kernels with polynomial basis functions of any order, both radial integrals may be performed in closed form, significantly speeding up the overall computation, and allowing us to separately study the convergence of the radial and contour integrations. For other kernels or bases, numerical integration is required, but we can now accurately predetermine the order of integration needed to guarantee any accuracy for the radial integration.

For the contour integrals, the case of extended line segments that intersect at a segment end point or interior point can now be handled. But the accelerating transformations introduce singularities at the intersection point that require an appropriate quadrature rule handle the singularity while retaining the original convergence properties. The orders of these singularities is currently under investigation. The fundamental formula for the method has recently been extended to non-coplanar elements, and some preliminary results have been obtained.