MLMDA-Based Direct Integral Equation Solver for Dielectric Scatterers

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Fast direct techniques for solving integral equations are rapidly gaining traction as viable alternatives to iterative schemes. Indeed, iterative methods converge slowly when applied to (intrinsically) ill-conditioned problem and grind to a halt when applied to problems involving many right-hand sides. Our group recently developed direct electric and combined field integral equation solvers that use the multilevel matrix decomposition algorithm (MLMDA) (a.k.a. butterfly schemes) to compress blocks of LU-factorized impedance matrices to attain $O(N\log^2 N)$ storage and CPU costs when applied to the analysis of scattering from 3D perfect electrically conducting surfaces (H. Guo，J. Hu and E. Michielssen, IEEE Radio Sci. Meeting, 2013 USNC-URSI, pp.91, Jul. 2013). Even though a rigorous proof of the method's storage and CPU requirements remains elusive, all experimental results obtained to date indicate that these solvers' memory and CPU requirements scale far more favorably than those of low-rank or adaptive cross approximation based solvers, which scale as $O(N^{\alpha}\log N)$ $(\alpha = 1.3 \sim 2.0)$ and $O(N^{\alpha}\log^{\beta} N)$ $(\alpha = 2.0 \sim 3.0, \beta \geq 1.0)$, respectively.

Here, we report on an extension of the above-referenced solver for homogeneous dielectric scatterers. Specifically, we describe an MLMDA based fast direct solver that applies to PMCHWT integral equations discretized using RWG basis functions. Like the PEC solver from which it derives, the new solver constructs a hierarchical block LU factorized impedance matrix by recursively operating on butterfly-compressed blocks of partial LU factors. In this process, the solver continuously constructs new butterfly-compressed representations of sums and products of butterfly-compressed matrices (H. Guo，J. Hu and E. Michielssen, IEEE Radio Sci. Meeting, 2014 USNC-URSI, pp.283, Jul. 2014), leveraging schemes that generalize randomized low rank and SVD approximation schemes to butterfly algebra (E. Liberty, F. Woolfe, P.G. Martinsson, et al., Proc. Natl. Acad. Sci., vol. 104, no. 51, pp. 20167-20172, 2007). Efficient factorization of the PMCHWT equations requires a reordering of the electric and magnetic current unknowns to guarantee that matrix blocks (of even size) can be hierarchically arranged into an MLMDA tree. The solver's computational efficiency is in line with that of the PEC solver that is based upon and will be demonstrated through numerical examples involving dielectric structures modeled in terms of millions of basis functions.