New Nonreflecting Boundary Conditions Based on Trefftz Approximations

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It is well known that finite difference (FD) and finite element solution of wave problems in unbounded domains must employ local, non-local, or PML-type artificial nonreflecting conditions at an exterior truncation boundary.

In this paper, we develop, in the FD context, local Trefftz absorbing conditions that were proposed earlier (Tsukerman, J Comp Phys, 211, 659–699, 2006; IEEE Trans. Magn., 41, 2206–2225, 2005) but remained largely unexplored.

As a model problem, we consider the scalar wave equation in the frequency domain: $\nabla \cdot \mu^{-1} \nabla u + k_0^2 \epsilon u = f$ in 2D or 3D. Sources f and scatterers are assumed to be confined to a finite domain. In 2D, this equation may describe the E-mode (TM) or, with the obvious change of notation, the H-mode (TE). But the methodology of this paper is general and applicable to more complex cases – 3D and vectorial. The wave equation is subject to the standard splitting of u into the incident and scattered fields with standard radiation boundary conditions (e.g. Sommerfeld) for the scattered field at infinity. Our task, however, is to replace these theoretical conditions with approximate but accurate and practical ones.

The proposed "Trefftz generator" of nonreflecting boundary conditions has two main ingredients: (i) a set of local Trefftz functions – *outgoing* waves satisfying the wave equation and approximating the solution near a given point on the exterior boundary, and (ii) a set of degrees of freedom (dof) – linear functionals such as the value of the solution and some of its derivatives at boundary points. Probing with Trefftz functions yields generalized FD schemes which collectively constitute a sparse linear system of equations.

We show that the proposed Trefftz generator reproduces classical Engquist-Majda (Math. Comp. 31, 629–651, 1977) and Bayliss-Turkel (Comm Pure & Appl Math., 33, 707–725, 1980) conditions and also opens up avenues for developing new conditions.

For canonical problems of 2D scattering from dielectric or perfectly conducting cylinders, convergence of order six on 9-point stencils is attained, and the relative error of the numerical solution is on the order of 10^{-5} – 10^{-8} with 10 – 20 grid points per vacuum wavelength. We are not aware of alternative methods yielding a similar level of accuracy on comparable grids. Extensions to 3D and vectorial problems are possible.