A Quasi-analytical Direct Transient Mie Solution for Spheres: the EM case

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Debye-Mie series solution is one of the most useful tools in time-harmonic analysis of electromagnetic scattering, and has found in extensive applications in both optics and electromagnetics. Their applications range from fields as diverse as light scattering from small particles to analysis of small antennas to photonics to biological applications. However, analytical solutions to scattering from a sphere only exist in the Fourier domain. Deriving their transient analogue is a challenge as it involves an inverse Fourier transform of the spherical Hankel functions (and their derivatives) that are convolved with inverse Fourier transforms of spherical Bessel functions (and their derivatives). Series expansion of these convolutions are highly oscillatory (therefore, poorly convergent) and unstable. Indeed, the literature on numerical computation of this convolution is very sparse.

Though no direct analytical transient Mie series has been discovered yet, research on transient scattering analysis using spherical harmonics is very attractive in providing a quasi-analytical solution. The methods investigated thus far (mainly for scalar problem) fall into two categories; (1) using inverse Fourier/Laplace transforms of Bessel and Hankel functions within a mode matching framework [Greengard et al., Journal of Computational Physics, 274, 191-207], and (2) using a spherical representation of the retarded potential Green's function [Li and Shanker, The Journal of the Acoustical Society of America, 135, 1676-1685]. The latter approach is to use vector tesseral harmonics as basis function for the currents in time domain integral equations. Thanks to a novel addition theorem for the time-domain dyadic Green's functions, the method to compute transient Mie scattering is both stable and rapidly convergent. This procedure results in an orthogonal, spatially-meshfree and singularity-free system, giving us a set of one dimensional Volterra Integral equations. Time-dependent multipole coefficients for each mode are obtained via a time marching procedure. During the conference, we would present (1) a stable spherical expansion for time domain dyadic Green's functions of both electric and magnetic types, (2) a set of reduced 1-D Volterra integral kernels that are solved with a higher order Galerkin scheme, and (3) several results demonstrating the accuracy, convergence and stability of the proposed methods.