Loop Subdivision Surfaces for the Generalized Method of Moments

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The Generalized Method of Moments (GMM) is a quasi-meshless method for discretizing boundary integral equations that partitions a scattering body into overlapping subdomains called patches (Dault et. al, *The Generalized Method of Moments for Electromagnetic Integral Equations*, IEEE Trans. on Antennas and Prop., Vol. 6, No. 2, 2014). Current descriptions on neighboring patches are stitched together using a partition of unity function. While GMM has been applied successfully to a range of boundary integral equations, the higher order surface descriptions used to describe non-flat scatterers may introduce significant complexity into the computation of matrix elements. Specifically, the "transition" maps between patches, which transform local coordinates on one patch into local coordinates on an overlapping patch, can be complicated nonlinear functions because they are compositions of the function describing the higher order surface and its inverse. For highly curved surfaces, these nonlinear transition functions complicate the process of integration, which in GMM is performed on a subtriangulation of the local patch coordinate domain. If images of subtriangles from a neighboring patch do not align with the subtriangles on the patch of interest, integrations can rapidly lose accuracy.

In this work, we utilize a different type of surface to represent the scattering geometry, the Loop subdivision surface (C. Loop, *Smooth Subdivision Surfaces Based on Triangles*, M.S. Thesis, 1987), which is defined as the limit of a sequence of recursive refinements of an initial low order triangular mesh. In the limit that the number of refinements goes to infinity, Loop subdivision produces a surface that is second order smooth in the sense that it possesses continuous first and second derivatives everywhere but a small number of vertices, where it is first order smooth. This property is particularly useful for GMM, which requires the absence of normal discontinuities interior to a patch. Furthermore, the limit surface may be expressed analytically in terms of a weighted set of quartic box splines parameterized in terms of barycentric coordinates of triangles on the initial mesh. Suitable reparameterization of these barycentric coordinates in terms of GMM local patch coordinates yields transition maps that are affine, and for which the image of a subtriangle on one patch always aligns with a subtriangle on an overlapping patch. Because the integration subdomains are conformal and the transition functions are linear, standard Gauss-Legendre quadrature rules can be used to integrate the partitions of unity and associated basis functions to high precision. The resulting method is not only more accurate but faster because of the ability to use lower order quadrature rules and the simplicity of evaluating the transition functions relative to previous implementations of GMM.