Comparison of Six, Four and Two-Vector Formalisms for Complex Media

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Advances in rapid prototyping using conductive-inkjet and 3D printing, for example, have made the fabrication of antennas and artificial complex media possible. These advances have prompted the need for alternative mathematical methods of analysis that are capable of handling anisotropic and bianisotropic media and also aid in better understanding of the physical nature of electromagnetic fields in complex media environments. As an example, a compact six-vector formalism for Maxwell's equations was developed (I.V. Lindell, A.H. Sihvola, and K. Suchy, J. of Electrom. Waves and Appl., 9, 7/8, 887-903, 1995) that aids in the mathematical manipulation and subsequent solution of problems involving complex media. This formulation can accommodate fully populated material tensors (i.e., all material tensor elements are non-zero), however, the analysis leads to a block 3x3 matrix that requires inversion. If the material property tensors of permittivity and permeability have no transverse-longitudinal or longitudinal-transverse components (i.e., the *xz*, *yz*, *zx*, *zy* elements are zero), then a four-vector approach can be advantageously employed leading to a block 2x2 matrix which is more easily inverted due to the reduced dimensionality compared to the six-vector approach. In addition, a more compact and physically-insightful form is often achieved for this specialized case of material tensors. If, furthermore, the material property tensors are restricted to gyrotropic media, then a two-vector technique can be readily sought that results in a block 1x1 matrix formulation, which substantially simplifies the analysis and leads to enhanced clarity of the physical nature of the electromagnetic fields.

The goals of this research are to first provide a brief overview of the fundamental details of the six-vector formalism for anisotropic media. The next goal is to develop a four-vector formalism for material tensors having no transverse-longitudinal or longitudinal-transverse elements. The final objective is to develop a two-vector formulation for the specialized case of gyrotropic material tensors. Advantages and drawbacks of each formulation will be discussed and the details of how to practically implement each method in the analysis of problems involving complex media will be provided. Future work will also be discussed, including the generalization of these results to bianisotropic media and the analysis of practical waveguiding and radiating structures in various complex media environments.

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