Green's Function for a Biaxial Medium using a Four-Vector Formalism

Andrew E. Bogle* and Michael J. Havrilla†
Air Force Institute of Technology, WPAFB, OH, 45433-7765, USA

The rapid prototyping of antennas and complex media propagation environments has been aided by recent advances in material fabrication capabilities; such as, conductive-inkjet and 3D printing. These advances have prompted the need for alternative mathematical methods of analysis, which more easily accommodate anisotropic and bianisotropic media while providing greater physical insight. For example, casting Maxwell's equations into a compact six-vector formalism aides in the mathematical manipulation, and subsequent solution, of problems involving complex media (I.V. Lindell, A.H. Sihvola, and K. Suchy, J. of Electrom. Waves and Appl., 9, 7/8, 887-903, 1955). This formulation accommodates fully-populated material tensors (i.e., material tensors where all elements are non-zero); however, the analysis does lead to a block 3x3 matrix that requires inversion.

Simple inversion formulas do not exist for this generic case; thus, a cofactor-based method is usually implemented, which is generally mathematically tedious and casts results into non-compact forms obscuring physical insight. If however, the material property tensors of permittivity and permeability have no transverse-longitudinal or longitudinal-transverse components (i.e., the *xz*, *yz*, *zx*, *zy* elements are zero), then a four-vector approach can be advantageously employed leading to a block 2x2 matrix; which is easily inverted using (H.C. Chen, Theory of Electrom. Waves, Thomson Press, 1983, pp. 19-23, 50-51), for example, due to the reduced dimensionality compared to the six-vector approach.

In this work, an electromagnetic four-vector formalism for anisotropic media is presented and compared to the six-vector formalism. Although the four-vector approach can be extended to bianisotropic media, the focus here is on anisotropic media, primarily, for the sake of brevity. It is shown that, if the material property tensors have a certain form, then the four-vector approach significantly reduces mathematical complexity and enhances physical insight due to the more compact formulation and reduced dimensionality. The Green's function for both electric and magnetic currents immersed in a biaxial anisotropic media is presented to show the effectiveness of the four-vector formalism and thus demonstrate its usefulness in the radiation and scattering analysis of antennas embedded in complex media environments.

† The views expressed in this article are those of the author and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the U.S. Government.