A Low Frequency Stabilized Convolution Quadrature Approach to Electromagnetic Scattering

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The convolution quadrature (CQ) method of temporal discretization has several properties that make it an ideal method for the construction of marching-on-in-time schemes (Q. Chen, P. Monk, X. Wang, and D. S. Weile, Commun. Comput. Phys, 11, 383–399, 2012). In particular, the temporal discretization they yield is always renders accurate spatial integrations without any sharp shadow transition, and they can represent the Green's functions in complicated, dispersive media with no special difficulty (X. Wang and D. S. Weile, IEEE Trans. Antenn. Propag., 59, 4651–4663, 2011). Finally, because of their close relationship with finite difference methods, they are ideal for the construction of boundary operators for FDTD (Y. Q. Lin and D. S. Weile, IEEE Trans. Antenn. Propag., 61, 2655–2663, 2014; S. Malevsky, E. Heyman, and R. Kastner, IEEE Trans. Antenn. Propag., 58, 3602–3609, 2010.)

Despite their high accuracy and broad applicability, however, CQ-based methods offer no special protection against low frequency instability caused by the well known separation of electric and magnetic effects of charges and currents at such frequencies. Indeed, the very broadening of the discretized Green's function that makes the CQ spatial integration so accurate also tends to increase numerical error in the actual convolution, giving rise to some unphysical low frequency currents, even in the absence of material dispersion or low frequency excitation. In this work, therefore, CQ based discretization of the time domain integral equations of electromagnetic is augmented with a loop-tree stabilization approach (R. A. Wildman and D. S. Weile, IEEE Trans. Antenn. Propag., 52, 3005–3011, 2004). The method works by first locating all of the purely solenoidal degrees of freedom in the mesh. As basis functions, these "loop" degrees of freedom have no net charge, and so can be used in integral equations to which no temporal derivative has been applied to eschew charge accumulation. As testing functions, loops annihilate the contribution of the scalar potential, which can thus be left uncomputed. Because this combination of careful field computation with careful testing both computes the elements of the convolution kernel more accurately than the conventional approach, it better avoids the generation of unphysical, low frequency modes. Moreover, the use of and undifferentiated form of the equation for the "tree" degrees of freedom and a differentiated form of the equation for the "loop" degrees of freedom ensures that these components are more scrupulously balanced at low frequencies. Numerical results will attest to the enhanced stability and accuracy of the proposed approach.