Conforming Testing of Surface Integral Equations for Penetrable Objects

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Electromagnetic scattering by a homogeneous penetrable object is a fundamental problem in many electromagnetic and electrical engineering radar, scattering and antenna applications. Surface integral equation method provides an elegant way to find solutions for such problems. For homogeneous penetrable bodies, the surface integral equations, however, can be formulated in many alternative ways. Different formulations are shown to have different numerical properties. For example, the accuracy of the solution and conditioning of the matrix depend strongly on the choice of the formulation (e.g., P. Ylä-Oijala, M. Taskinen and S. Järvenpää, Radio Science, 40, RS6002, 2005). Recently, it has been observed that the conventional discretization technique based on the Galerkin's method and divergence conforming basis and testing functions (e.g., Rao-Wilton-Glisson functions), does not necessarily provide the most optimal testing procedure for all formulations. In particular, for the integral equations of the second Galerkin's method may not lead to optimally converging solutions.

A general requirement for converging solutions is that the discretization procedure is "conforming", meaning that the finite element spaces of the basis and testing functions have to be defined so that they form finite dimensional spaces for the domain space of the associated integral operator and dual of its range space. If these conditions are not satisfied, there is no guarantee that the numerical solution procedure converges towards the correct solution in the norm of the numerical solution space as the mesh density is increased. The importance of conforming discretizations has been demonstrated for both perfectly conducting objects (K. Cools, F.P. Andriulli, D. De Zutter, and E. Michielssen, IEEE Antennas Wirel. Propag. Lett., 10, 528-531, 2011) and homogeneous penetrable objects (P. Ylä-Oijala, S.P. Kiminki, K. Cools, F.P. Andriulli, and S. Järvenpää, Intern. J. Num. Model. Electronic Networks, Devices and Fields, 25, 525-540, 2012).

In this presentation we perform both theoretical and numerical analysis of the accuracy of the solution of various surface integral equation formulations for homogeneous dielectric bodies. We focus on the near field accuracy and study how well the formulations satisfy the electromagnetic boundary conditions on the surface of the object. Our numerical experiments clearly demonstrate the necessity of conforming testing, in particular, for high contrast non-smooth objects. We also show that some popular formulations, like the Müller formulation, may lead to very high error in the near field, if the equations are not discretized properly.