Gabor Deconvolution: Attenuation Function Estimation based on Frequency-dependent Q

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Gabor deconvolution has been applied in seismic imaging in order to compensate for non-stationary effects encountered during wave propagation (G. F. Margrave, et al., Geophysics, 76, 15-30, 2011). We previously applied this method to signals collected during microwave breast imaging (K. Y. Liu, et al., URSI GASS, 1-4, 2014) for the purpose of compensating for the frequency-dependent attenuation of wave propagation in lossy and dispersive tissues; and improving the signal resolution by removing the excitation pulse in the recorded signal. The aim of this study is to investigate a frequency-dependent Q model that is able to more accurately describe the wave attenuation than the existing constant Q approximation, in order to provide us with better estimation results.

In Gabor deconvolution, the Gabor transform of the received signal, $S_g(t, f)$, is given as

$$S_g(t,f) \approx W(f)B(t,f)R_g(t,f) \tag{1}$$

where t is the time, f is the frequency, $W(f)$ is the Fourier transform of the source pulse, $B(t, f)$ is the function that accounts for the attenuation and dispersion, and $R_g(t, f)$ is the Gabor transform of the reflectivity. In order to recover $R_g(t, f)$, we need to estimate the source pulse and the attenuation function from the received signal. With the condition of minimum-phase source pulse and wave attenuation process, we only need to consider the magnitude spectrum because the phase is computable as the Hilbert transform of the natural logarithm of the magnitude.

In seismic imaging, it is widely accepted that the wave propagation and attenuation in geological media can be approximated by a constant Q model (E. Kjartansson, Geophys. Res., 84, 4737-4784, 1979). Q is defined as the ratio between the stored energy and the lost energy during a wave cycle (A. R. V. Hippel, 1959). Defining the amplitude spectrum of the attenuation function as

$$\left|B(t,f)\right| = \exp(-\pi\left|f\right|t/Q) \tag{2}$$

suggests that $|B(t, f)|$ can be estimated by smoothing the Gabor magnitude spectrum along hyperbolic paths defined by tf=constant. However, this constant Q approximation may not be valid in microwave breast imaging. We calculate the Q for various body tissues and immersion media over microwave frequencies, and results indicate that Q varies with frequency. Discrepancy is also observed in the shape of hyperbolas constructed with the constant Q and the frequency-dependent Q. To investigate the impact of this discrepancy on the recovered reflectivity, we use the hyperbolas calculated from frequency-dependent Q to estimate the attenuation function and compare the results with those obtained from constant Q estimation. Using plane wave propagation in infiltrated fat as an example, the results indicate that smaller estimation errors are obtained from the frequency-dependent Q model. However, the difference between the approximated and the exact hyperbolas does not significantly affect the estimate of the reflectivity. Testing with additional materials permits us to define the limits, within which Gabor deconvolution is still able to give a reasonable estimate of the reflectivity with the constant Q approximation. As well, we are interested in investigating a new parameter calculated from the higher order derivative of the Q definition that has better constancy over the microwave frequencies than the Q does.