# The Meaning of a Recent Single Scattering Approximation

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In a recent URSI presentation we demonstrated the merits of splitting the current, due to an incident filed upon a conducting body or surface, into the sum of a single and multiple scattering parts. The crucial step in this approach was insuring that the complete single scattering part of the current was accounted for. Contrary to previous work (particularly in the rough surface scattering area), we found the single scattering current to contain more than the Kirchhoff approximation. However, the source and meaning this new result was not pursued due to the need to complete the full single/multiple scatter formalism. The purpose of this presentation is to delve into this new single scattering result and identify its physical meaning.

The single scattering or local boundary condition approximation amounts to assuming that the unknown current, say, at a point on a rough surface is not dependent upon the current at any other point on the surface. While this is a very simple definition, it is not always easy to pull the single scattering solution out of a second-kind boundary integral equation. For example, in rough surface scattering from a perfectly conducting surface, the Kirchhoff approximation is frequently taken as the single scattering solution and this does work very well for a class of surfaces but not all. However, for another class of surfaces one must employ a boundary perturbation approach to obtain an accurate solution. Interestingly enough this solution obeys the definition of a single scattering or local approximation but seems to require cooperative scattering from two points on the rough surface. In addition, this solution seems to come from the second Neumann iterate of the governing integral equation. This paradox raises the question of what truly is the single scattering or local approximate solution of a second kind boundary integral equation.

It is relatively easy to obtain the true single scattering current by means of simple mathematical manipulations but they give no insight into the physical source or meaning of the result. In the scalar case one finds that the single scattering part of the current is given by the following form; $J_o(x)[1-\int G(x,x')dx']^{-1}$. Here, $J_o(x)$ is the Kirchhoff approximation and $G(x,x')$ is the kernel of the integral equation which, in the scalar case, is related to the normal derivative the free space Green function evaluated on the scattering surface. The Kirchhoff term provides the correct single scatter high frequency behavior of the current for a sufficiently gently curving body. The physics associated with the bracketed term is more difficult to determine. One can attempt to approximate the integral but this is difficult and not particularly insightful. A more straightforward approach involves recognizing that the bracketed term comes from replacing the current $J(x')$ by $J(x)$ in the integral equation and then solving for the (single scatter) current. The meaning of this replacement is that we have assumed the current on the body is everywhere the same and this is what takes place for the case of scattering by a conducting body in a very low frequency field. Thus, the bracketed term represents the low frequency behavior of the current. We will explain this result in detail and show that it is also the source of the so-called two scale model for scattering from rough surfaces that has been so useful in microwave remote sensing.