Error Estimation and Adaptive Refinement for Integral Equations

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Surface integral equation formulations are popular approaches for electromagnetic radiation and scattering problems. Unlike differential equation formulations, which are routinely implemented using adaptive refinement procedures to assist the analysis, there appears to be no widespread use of adaptive refinement techniques applied to integral equations. Adaptive refinement consists of adjusting cell sizes (*h*-refinement) or representation orders (*p*-refinement) to achieve a more accurate and efficient solution without user intervention. The approach reduces the need for user expertise and provides a more systematic solution procedure.

One of the major ingredients necessary for adaptive refinement is an error estimator. Error estimates are often based on residual field calculations (how well an electromagnetic boundary condition other than that used within the equation itself is satisfied), or an assessment of the extent of a field, current, or charge discontinuity. These are explicit estimators, in that they return an estimate for a particular approximate solution. Implicit estimators typically divide the problem domain into small subdomains and re-solve the problem on each using smaller cells or higher degree representations. The error estimate is based on the extent to which the original solution changes on each subdomain, and does not require an approximate solution for the whole problem as starting point.

Previous research has concentrated on 2D problems involving conducting targets. Saeed and Peterson evaluated several 2D estimators that correctly identify the region of the domain with the highest error, and are potentially useful for adaptive refinement. Residual estimators are robust but impose a relatively expensive computational cost equal to that of constructing the system matrix. Charge discontinuity estimators, while less robust, are found to be reasonably accurate in practice and are relatively inexpensive.

In this presentation the performance of several error estimators will be compared for 3D surface integral equation formulations, discretized using the method of moments and standard representations such as Rao-Wilton-Glisson basis functions. A suite of test targets with perturbations to induce localized errors will be employed. The performance of the estimators and their relative cost will be highlighted. Simple adaptive refinement algorithms based on the best estimators will be illustrated.