Low-Frequency Stable Internally Combined Volume-Surface Integral Equation for 3D High-Contrast Scatterers

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Volume-surface integral equations (VSIEs) are often used to analyze electromagnetic scattering by piecewise homogeneous dielectric objects (Usner et al., *IEEE Trans. Antennas Propagat.,* vol. 54, pp. 68-75, 2006). Unfortunately, when these objects include high-contrast (HC) regions, viz. regions with permittivities that are much higher than those of their surrounding media, discretization of many VSIEs results in ill-conditioned systems of equations. In addition, VSIEs invariably suffer from low-frequency (LF) breakdown: when the mesh that discretizes the scatterer contains elements with dimensions that are much smaller than the wavelength, their discretization results in ill-conditioned systems of equations. Both HC and LF breakdown phenomena severely limit the usefulness of VSIEs in the analysis of bio-electromagnetic and geophysical applications. Previously, we developed a novel internally combined volume-surface integral equation (ICVSIE) and showed that it is immune from HC and LF breakdown when used to analyze TE scattering from various 2D inhomogeneous cylinders (Gomez et al., *USNC/URSI Nat. Radio Sci. Meeting*, 2013).

Here, we extend our previous work by developing and implementing a 3D ICVSIE and associated solver. The new 3D ICVSIE neither suffers from HC nor LF breakdown phenomena when used to analyze scattering from general 3D high-contrast objects. The ICVSIE is constructed by: (i) subdividing the object into regions each having a small (controlled) ratio of maximum to minimum permittivity, (ii) wrapping each region in equivalent electric and magnetic surface currents and invoking surface equivalence principles to artificially increase the effective permittivity of the “background medium” in which each region’s polarization currents radiate, and (iii) assembling the ICVSIE system of equations consisting of coupled Muller combined field integral equations (CFIEs) and novel combined VIEs that are constructed by adding to standard VIEs the electric field generated by exterior surface currents on the boundary of each region to the VIE. Numerical results obtained by analyzing electromagnetic scattering from various 3D high-contrast objects confirm that the ICVSIE does not suffer from either HC or LF breakdown. In other words, discretization of the ICVSIE yields matrices with condition numbers that are unaffected by the scatterer’s maximum permittivity and electrical size.