Efficient Parallelization of the FETI-DP Algorithm for Large-Scale Electromagnetic Simulation

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For decades, the finite-element method (FEM) has been proven to be a versatile approach in modeling complicated materials/systems for its strong adaptability to complex geometries and high numerical accuracy (J.-M. Jin, *The Finite Element Method in Electromagnetics*. 3rd ed., Hoboken, NJ: Wiley, 2014). Unfortunately, a volumetric discretization, as required by the FEM, would easily yield a linear system with millions or even billions of unknowns for modern engineering applications such as antenna array analysis. The domain decomposition-based FEMs have been therefore developed to enable large-scale electromagnetic (EM) simulations on parallelized computer clusters, among which the dual-primal finite-element tearing and interconnecting (FETI-DP) algorithm for EM analysis is highly powerful because of its numerical stability and scalability (Y.-J. Li and J.-M. Jin, *J. Comput. Phys.*, 228, 3255-3267, 2009).

The FETI-DP algorithm divides an entire computational domain into many non-overlapping subdomains and enforces Robin transmission conditions at the subdomain interfaces to form an equivalent order-reduced interface problem. To accelerate the iterative convergence of the interface problem, a global coarse system, which relates only primal unknowns at the corner edges of the subdomain interfaces, is constructed and solved using a direct solver. Due to the relatively poor parallel performance of direct solvers on distributed computing systems, the achieved parallel efficiency with an increasing number of computation nodes for the current FETI-DP implementation is compromised, especially when simulating large-scale complex applications involving a huge number of corner edges.

In this work, we present an efficient parallelization of the FETI-DP algorithm for large-scale EM simulation. In particular, we describe the implementation of a parallelized Krylov subspace method for an iterative solution of the global coarse system to remove the bottleneck for the parallelization of the FETI-DP algorithm. Based on the sparse pattern of the global coarse system, we further develop an efficient sparse preconditioner to improve the convergence rate of the iterative solution. We then study the iterative stop criteria for the global coarse system and the interface problem in terms of numerical accuracy and solution time. Finally, we present several numerical examples to demonstrate the accuracy, scalability, and capability of our new parallel implementation of the FETI-DP algorithm for EM modeling of large-scale complex objects and materials.