Novel Error-Control Methodology for Finite Difference and Finite Element based Electrostatic Green's Function Computation in Inhomogeneous Substrates

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Electrostatic analysis of complex 3-D structures represents an indispensable design optimization tool and essential verification stage in modern electronic design automation of integrated circuit chips and packages. Maxwell capacitance matrix of multi-conductor geometries embedded in inhomogeneous substrates is among the primary quantities that an electrostatic field solver produces. Accurate knowledge of Maxwell capacitance matrix is crucial for signal integrity characterization and quantifying critical performance-related circuit features such as speed and functionality. Integral equation formulation for capacitance extraction is favored over its differential equation counterpart since its method-of-moments (MoM) or locally-corrected Nyström (L. F. Canino, J. J. Ottusch, M. A. Stalzer, J. L. Visher, and S. M. Wandzura, J. Comput. Phys., vol. 146, no. 2, pp. 627-663, 1998) matrix representation only involves discretizing the surface of the conductor upon the availability of the Green's function for the background medium. A finite difference method (FDM) based and high-order finite element method (HO-FEM) based electrostatic Green's function computation in planar stratified media have been introduced in (A. Cangellaris and L. Yang, IEEE Trans. Magn., vol. 37, no. 5, pp. 3133-3136, 2001) and (M. Al-Qedra and V. Okhmatovski, IEEE Int. Symp. on Antennas and Propagation and USNC-URSI Radio Science Meeting, pp. 189, 2013) respectively with several practical numerical examples. In this work, we extend both the FDM and the HO-FEM based techniques for electrostatic Green's function computation in planar stratified media to include mathematical formulation that allow for quantitative error analysis. The formulation begins with general expression of spectral domain Green's function at any point in the layered media as a sum of primary (in vicinity of source) and secondary field terms. The exact evaluation of spectral domain Green's function is composed of numerically calculated term superposed with error term. Using Taylor expansion for the exponentials carrying the spectral variable and with the knowledge of the utilized numerical scheme (FDM or HO-FEM), the Taylor expansion is truncated to a finite set of polynomials corresponding to the error function. Taking the inverse Fourier-Bessel transform of the error function yields the error function with respect to location and grid size. In addition we provide three numerical comparison studies. First, developed numerical method is used to simulate for structures having known analytical solutions. Another study consists of refining the mesh (computational domain) until convergence of result is achieved according to a predefined accuracy. Moreover, the computational method is benchmarked with other gold standard software.