An Uncertainty Principle in Electromagnetic Homogenization

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Nontrivial magnetic response of periodic structures composed of intrinsically nonmagnetic constituents is now a well-established but still intriguing phenomenon; it plays a critical role in negative refraction, cloaking and other unusual effects. Much attention has been devoted to optimal (in some sense) design, whereby the magnetism would be enhanced and engineered to fall in a desirable range for specific applications – absorption, cloaking, lensing, etc. However, we show both analytically and numerically that this artificial magnetism has limitations: the stronger the magnetic response (as measured by the deviation of the effective permeability tensor from identity), the less accurate ("certain") predictions of the effective medium theory are. We call this the *uncertainty principle* for the effective parameters of metamaterials. In practice, there is still room for engineering design, but the trade-offs between magnetic response and the accuracy of homogenization must be noted.

A basis of our analysis is the recently developed non-asymptotic homogenization theory for periodic electromagnetic structures (*Proc. R. Soc. A*, 2014.0245). The homogenization procedure consists in (i) finding suitable approximations of fine-level (i.e. sub-cell) and coarse-level (coarser than the lattice cell size) fields, and (ii) establishing a constitutive relationship between the pairs of coarse fields. No assumptions other than the intrinsic linearity of the constituent materials of the structure are made; in particular, anisotropy and magnetoelectric coupling may exist, and the lattice cell size does not need to be vanishingly small.

As an instructive example, we consider the triangular lattice of cylindrical air holes in a dielectric host, as investigated previously by Pei & Huang (JOSA B, 2334–2338, 2012). This example is interesting because it exhibits a particularly high level of isotropy around the Γ-point in the second photonic band. Even in this highly isotropic case the uncertainty principle remains valid. First, isotropy with respect to the Bloch wavenumber is not accompanied by isotropy of the Bloch impedance; second, surface waves play a significant role at shorter wavelengths.