Hierarchical Vector Bases for 3D Problems

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Hierarchical curl- and divergence-conforming vector bases for two- and three-dimensional cells were derived and discussed in a four paper series published by Graglia, Peterson *et al.* in the *IEEE Trans. Antennas Propagation* ([1] vol. 59, pp. 950-959, 2011; [2] vol. 59, pp. 2766-2773, 2011; [3] vol. 60, pp. 3314-3324, 2012; [4] vol. 60, pp. 5215-5227, 2012). Relatively few results for 3D problems are reported in these papers due to an emphasis on (a) the procedures used to derive the hierarchical vector bases and (b) the linear independence of the basis functions. Conversely, results for two-dimensional problems obtained using hierarchical vector bases functions on triangular and quadrilateral cells are reported in two short papers by Peterson and Graglia ([5] "Scale factors and matrix conditioning associated with triangular-cell hierarchical vector basis functions," *IEEE AWPL*, vol. 9, pp. 40-43, 2010. [6] "Evaluation of hierarchical vector basis functions for quadrilateral cells" *IEEE Trans. Mag.*, vol. 47, no. 5, pp. 1190-1193, 2011).

In addition, *additive* singular hierarchical bases were derived in order to numerically model the unbounded behavior of the fields in the vicinity of sharp metallic or penetrable wedges. To date, the available singular bases are those for two-dimensional (triangular or quadrilateral) cells, and their development and supporting results are available in four papers authored by Graglia *et al.* ([7] *IEEE Trans. Antennas Propagat.*, vol. 61, pp. 3674-3692, 2013; [8] *IEEE Trans. Antennas Propagat.*, vol. 62, pp. 3632-3644, 2014; [9] *Electromagnetics*, Special Issue on Finite Elements for Microwave Engineering, vol. 34, pp. 171-198, 2014; [10] *IEEE AWPL*, accepted July 2014, available on IEEE Xplore - early access).

In this work, we review the development of hierarchical vector bases and report additional results obtained using them for simple 3D canonical problems. Since in contrast to interpolatory bases, hierarchical bases often exhibit poor linear independence as the order of the representation is increased, resulting in an ill-conditioned system of equations, in the presentation we will consider in particular the linear independence (or ill-conditioning) issue. A procedure for deriving singular additive curl-conforming bases functions able to model wedge-type singularities with the most common three-dimensional cells will also be outlined.