Evaluation of Sommerfeld Integrals Using Adaptive Filon-Type Integration

G. J. Burke
Lawrence Livermore National Laboratory, Livermore, CA 94550
email: burke2@llnl.gov

Modeling antennas in a layered medium by the method of moments solution of integral equations requires repeated evaluation of the Green's function for the medium. Much work has been done on the rapid evaluation of these Sommerfeld integrals. A commonly used method is the Discrete Complex Image Method, but numerical evaluation of the Sommerfeld integrals is sometimes desirable for direct control of the error and arbitrary source and evaluation point locations. Direct integration together with an interpolation/table-lookup scheme may be fast enough for the full MoM solution.

The integrals have the form $S_\nu(\rho, z, z') = \int_0^\infty G(k_\rho, z, z') J_\nu(k_\rho \rho) k_\rho \, dk_\rho$ where G is the spectral domain Green's function and J_ν is the Bessel function of order ν. The integrals are often evaluated by integration of k_ρ on the real axis using a method such as weighted averages to accelerate convergence to infinity, while numerically integrating early k_ρ on a contour deformed into the complex plane to avoid singularities. When the radial distance ρ is large there can be many oscillations due to $J_\nu(k_\rho \rho)$ on this numerical integration contour making evaluation difficult. A method for evaluating Fourier integrals with rapidly oscillating integrands was developed by L.N.G. Filon (Proc. Roy. Soc. Edinburgh 49, 38-47, 1928) and the adaptation to Hankel transforms was done by R. Barakat and E. Parshall (Appl. Math. Lett. Vol. 9, No. 5, pp. 21-26, 1996). Based on Simpson's rule, the slowly varying function G is approximated with a quadratic over a sub-interval of the range and the terms multiplying J_ν are integrated with approximations independent of the oscillation rate. Subintervals are then summed over the full range. Paralleling Filon, for J_0 we integrate by parts twice to get the integrals $I_1(\rho, z) = \int_0^z x J_1(\rho x) \, dx$ and $I_2(\rho, z) = \int_0^z I_1(\rho, x) \, dx$ that are evaluated with series and asymptotic approximations. Similar integrals are obtained with J_1.

An adaptive procedure is implemented with a standard method for Simpson's rule, using two panels of Simpson over five points and comparing the result using points (1,3,5) with (1,2,3) plus (3,4,5) as a measure of how well G is approximated by a quadratic. If the test is not passed the interval is halved, resulting in refinement of the intervals near singularities while taking large steps in smoother regions. When the interval size gets less than about a period of J_ν an optimum integration method such as Gauss-Legendre can be used for the interval.

Results are validated for the Sommerfeld identity and against asymptotic and guided wave results.

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344