Some Investigations Toward Closed-Form Solutions of Sommerfeld Integrals

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Sommerfeld integrals, that appear in problems related to electromagnetic wave propagation through layered media, have been traditionally evaluated by locating the poles of the integrand and accounting for their corresponding residue contributions. For multilayer problems, implementation of this procedure could potentially become computationally inefficient because the number of poles with significant residue contributions increases with frequency or equivalently the electrical thickness of the individual layers. This observation suggests that the well-known and popular discrete complex image method (DCIM) could also need significant modifications to its conventional algorithmic implementation for electrically thick layers.

For source and observation points with large lateral separations, and located close to or at the interface of two layers in a multilayer topology, the convergence of the integrands in the Sommerfeld integral degrades due to strong oscillatory behavior produced by the Bessel functions. In such cases, the Sommerfeld integrals are evaluated in closed form using asymptotic techniques which facilitate in rapid calculations in a moment method solution for microstrip arrays. The main feature of such asymptotic solutions is facilitated by using large argument form of the Bessel (or Hankel) functions. The attendant limitation of such an approach is that while mutual coupling can readily be calculated, computation of the input impedances may not be accurate enough because of the use of such approximate (asymptotic) techniques.

Recent investigations, primarily focussed around single-layer microstrip topologies, have utilized closed-form evaluation of a portion of the Sommerfeld integral without any approximation of the Bessel (or Hankel) functions, plus direct numerical integration of a finite integral [D. Chatterjee, S. M. Rao & M. S. Kluskens, *Proc.* 2013 *URSI EMTS*, pp. 381-384]. The direct numerical integration is performed along the contour in the upper-half plane where there are no pole contributions to the Sommerfeld integral. While the proposed method has been found to compare well with direct asymptotic solutions, a possible alternative could be to evaluate the finite integral via the method of stationary phase for electrically large separations. This, together with the closed form evaluation of the Sommerfeld integral "tail" suggests the possibility of improved closed-form solutions for the Sommerfeld integral as compared to the direct asymptotic formulations. This details of this proposed new method will be discussed at the presentation.