Exact Scattering by Concave Parabolic Cylinders and Paraboloids

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A concave parabolic cylinder is not a truly open surface, but the limit case of a closed surface: an elliptic cylinder when the distance between the interfocal lines tends to infinity while the difference between the length of the major axis and the interfocal distance remains finite. Similarly, a concave paraboloid of revolution is the limit case of a prolate spheroid when the distance between its foci tends to infinity while the difference between the length of the major axis and the interfocal distance remains finite. In either case, the only possible primary plane wave must propagate symmetrically with respect to the concave surface and must be accompanied everywhere by a reflection of the wave at the concave surface itself. Analytically, this reflected portion of the incident field takes the form of the product of the primary plane wave times a special function, that is a Fresnel integral in the case of a parabolic cylinder and an exponential integral in the case of a paraboloid. Thus, the incident field consists of the sum of these two waves. The reflected field also consists of two components: a plane wave propagating in the direction opposite to the direction of the incident plane wave, and the product of this plane wave times a special function, that is again a Fresnel integral or an exponential integral, but with different arguments from those of the functions occurring in the incident field.

The analysis was first performed for the convex parabolic cylinder by Lamb (*Proc. London Math. Soc.*, vol. 4, pp. 190-203, 1907), who indicated that a similar technique could be used for the convex paraboloid. However, Lamb was unable to solve the boundary-value problem for the concave structures and, in fact, stated that his method was not suitable for such cases. Recently, Uslenghi obtained the solution to scattering by the concave parabolic cylinder by matching incident and reflected fields at the focal line (*IEEE Antennas and Wireless Propag. Lett.*, vol. 11, pp. 412-422, 2012), and to scattering by the concave paraboloid in the acoustical case by matching incident and reflected fields at the focus (*Atti Acc. Scienze Torino*, Nov. 2013, in press; *Proc. Days on Diffraction*, St. Petersburg, Russia, May 2014).

In this work, the history of the problem is reviewed and the novel solution of electromagnetic scattering by a concave metallic paraboloid is presented. Also, the energy integrability condition in the symmetry plane of the parabolic cylinder and at the symmetry axis of the paraboloid is discussed. The analysis is conducted in the phasor domain with the time-dependence factor *exp(jωt)* omitted throughout.