

# Optimization of 3D Multilevel Non-Uniform Grid Back-Propagation Algorithm by Using Modified Oblate Spheroidal Coordinates

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A large class of source imaging methods is based on the radiated field back-propagation, often referred to as antenna holography, which has been widely used for large reflector antenna analysis. In these methods, the field is measured in either the near- or far-field zone of the source, by using a probe whose influence on the measured data is compensated. The measured field is propagated back to the source surface and the resulting reconstructed distribution is compared to the desired one. The comparison allows for the localization of distortions in the source's topography or current distribution. Although back-propagation does not accurately recover the original source distribution, it provides a good approximation of the radiating subspace of source distributions, up to an error due to the truncation of the measurement surface.

As a representative example, we analyze parabolic reflector geometries using measurements on a planar surface, at a single frequency. For this case, it is convenient to use the Rayleigh-Sommerfeld integral formulation for back-propagation. Assuming both source and measurement surfaces are discretized by using localized basis functions, the direct numerical evaluation of the back-propagation integral's discretized form is characterized by a computational complexity (CC) of  $O(N^4)$ , where  $N = kR$  is a large parameter, with  $R$  being the radius of the smallest sphere circumscribing the measurement domain, and  $k$  - the wavenumber. Aiming to reduce the CC down to  $O(N^2 \log N)$ , while still being able to reconstruct the field on a concave surface of the reflector, we propose the use of the Multilevel Non-Uniform Grid (MLNG) algorithm (A. Gergel, Y. Brick, and A. Boag, ICEAA 2014, pp. 57-59). The MLNG scheme relies on the sampling of the partial phase- and amplitude-compensated field contributions on optimal sampling grids. The integration (measurement) domain is decomposed hierarchically into smaller sub-domains and non-uniform sampling can be used in a multilevel divide-and-conquer type procedure to capture partial contributions to the overall integral.

In this work, we present an MLNG algorithm, for fast field integral evaluation, which makes use of the optimal volumetric sampling schemes in oblate spheroidal coordinates. For this coordinate system, analytical expressions for the phase- and amplitude-compensation and restoration factors are developed, together with optimal sampling criteria for the compensated fields. The performance in terms of accuracy and CC is compared to that achieved with the conventional spherical non-uniform grids.