Analytical Approach to Modeling Flat Lenses with Continuously Graded Profiles

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In this work we present an analytical approach to deriving the field solutions for a class of flat lenses that have attracted the attentions of antenna designers and researchers alike. The lens designs typically consist of a number of layers of graded index dielectrics, whose properties may vary in both the radial and longitudinal directions. The fields propagating in the longitudinal direction through the central layer primarily contribute to the bulk of the phase, while the side layers act as matching layers and help reduce the reflections originating at the interfaces of the middle layer. We model such lenses as compact composites with material properties characterized by continuous permittivity and permeability functions, which tend asymptotically to unity at the boundaries of the composite cylinder.

The designs of flat lenses using Transformation Optics (TO) with metamaterials (MTMs) have been proposed in a number of recent publications (e.g. R. Yang, W. Tang, and Y. Hao, Opt. Express, 19, 12348 - 12355, 2011). However, the composites required to implement such MTM-based designs may be difficult to realize in practice, especially when the required values of relative permeability and permittivity are either less than unity or are very high. The problems in realizing MTMs with the requisite values of permeabilities required by the TO-based designs can be circumvented by by setting, for instance, $\mu_r = 1$ and using only dielectric materials that approximate the original design, albeit at the cost of compromising its performance. While the Ray Optics (RO) approach to lens designs avoids the difficulties encountered with the TO designs mentioned above, it lacks the same flexibility to control the phase and amplitude of the fields within the lens structure. An approach to mitigating the shortcomings of both the TO and RO designs is to employ the Field Transformation (FT) method, introduced recently in (S. Jain, M. Abdel-Mageed and R. Mittra, IEEE Ant. Wirel. Propag. Lett., 12, 777 - 780, 2013). Regardless of which approach is used to carry out the design of a flat lens, the field solution in such a lens is typically derived by employing a purely numerical approach.

In contrast to this, in this work we investigate the possibility of identifying a class of flat lens designs, with profiles that lend themselves to analytical or quasi-analytical (e.g., based on the WKB method) solutions of the field equations. Our principal motivation for pursuing the analytical or quasi-analytical avenues for modeling flat lenses is that a detailed knowledge of analytical structure of the field solutions can potentially provide not only a better understanding of the lens performance, but also guidelines for either improving the designs or for developing entirely new/improved designs. To illustrate the proposed procedure, the paper will provide the analytical solutions for the electric and magnetic fields for a special class of composite designs. Techniques for extending the proposed approach to the analysis of existing flat lens designs will be included in the presentation.