Three Representations of Signal Propagation in a Random Waveguide

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Quite often in applications one has the need to model signal propagation in inhomogeneous media with irregular boundaries. Our interest is to understand the nature of volume-surface scattering interactions. To highlight this we consider a random medium layer with perfectly reflecting rough boundaries. All fluctuations are zero-mean stationary processes independent of each other. A point source is embedded in the random medium, and we are interested in the field structure thus generated.

Assuming that the boundary fluctuations are small and smooth the problem is formulated as an integral equation for the Green's function where volumetric and surface fluctuations are represented in a unified manner. Noting that that the problem is translationally invariant and isotropic in azimuth we formulate the mean Green's function as that of a homogeneous slab medium with planar boundaries. We use the Weyl representation for the Green's function where the unknowns of the problem are the effective propagation constant and the mean reflection coefficient. By employing certain operators on the Dyson equation we evaluate the unknowns of the problem. We hence find that the effective propagation constant is complex implying signal attenuation. It is independent of boundary fluctuations and primarily determined by the spectrum of volumetric fluctuations. On the other hand, the surface scattering coefficients depend on both volumetric and surface fluctuations. Furthermore, in contrast to the Dirichlet boundary the volume-surface interactions in the reflection coefficient of the Neumann boundary are far more significant.

By expanding the singular term of the mean Green's function into a binomial series we carry out the integration of each term in the small wave length limit. This leads to the ray representation of the Green's function. Here we see that the ray undergoes attenuation as it propagates in the random medium and loses energy every time it reflects from the boundaries.

A third representation is obtained by identifying the poles of the Weyl integral and expressing it as a sum of the residues. These are the wave guide modes whose wave numbers are found to be complex. We find that the resulting attenuation is determined by the spectra of all the three fluctuations of the problem. This mode representation is very useful for propagation loss calculations when the layer thickness is small. In contrast, ray representation is preferable when the layer thickness is large. Our three representations of the Green's function give us three different perspectives of volume surface interactions.