Localized Monochromatic and Pulsed Waves in Hyperbolic Media

Ioannis M. Besieris [(1)*] and Amr M. Shaarawi [(2)]
(1) The Bradley Department of Electrical and Computer Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24060 USA
(2) Department of Physics, The American University in Cairo, P.O. Box 74, New Cairo 11835 Egypt

Consider a uniaxially anisotropic material with diagonal electric permittivity and magnetic permeability tensor elements $\varepsilon_{xx}, \varepsilon_{yy} = \varepsilon_{xx}, \varepsilon_{zz}$ and $\mu_{xx}, \mu_{yy} = \mu_{xx}, \mu_{zz}$, respectively. If all four elements are positive, the dispersion relation is described by an ellipsoid. Within a certain frequency band, however, it may turn out that not all diagonal elements are positive and the dispersion relation is a hyperboloid. Under these conditions, the material is referred to as a *hyperbolic medium*. For simplicity, the discussion will be confined to a nonmagnetic material. Source-free, transverse magnetic electromagnetic fields in the frequency domain are expressed in terms of an appropriately defined Hertz vector potential $\vec{\Pi}(\vec{r},\omega) = \tilde{\Pi}_e(\vec{r},\omega)\hat{z}$ governed by the equation

$$\left(\nabla_t^2 + \frac{\varepsilon_{zz}}{\varepsilon_{xx}}\frac{\partial^2}{\partial z^2} + \frac{\varepsilon_{zz}}{\varepsilon_0}k^2\right)\tilde{\Pi}_e(\vec{r},\omega) = 0; k \equiv \omega / c.$$

For $\varepsilon_{xx} < 0$ and $\varepsilon_{zz} > 0$, the expression above is a de Broglie-like equation, with the coordinate z being timelike. On the other hand, for $\varepsilon_{xx} > 0$ and $\varepsilon_{zz} < 0$, one has a Klein-Gordon-like equation, again with a timelike z coordinate. For both cases, large classes of spatially localized solutions $\tilde{\Pi}_e(\vec{r},\omega)$ are available. A parabolic approximation of the de Broglie-like equation along the y direction yields an equation analogous to that arising in the study of bidispersion. Using hyperbolic rotations, a broad class of skewed, nonspreading, "accelerating" Airy solutions can be obtained.

Suppose the permittivity tensor elements are constant within a narrow frequency regime. Then, approximately, one has

$$\left(\nabla_t^2 - \frac{\varepsilon_{zz}}{|\varepsilon_{xx}|}\frac{\partial^2}{\partial z^2} - \frac{\varepsilon_{zz}}{\varepsilon_0}\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\Pi_e(\vec{r},t) = 0$$

in the time domain for $\varepsilon_{xx} < 0$ and $\varepsilon_{zz} > 0$. A large class of spatiotemporally localized luminal, subluminal and superluminal pulsed solutions to this equation can be derived. These solutions differ substantially from the analogous ones in isotropic free space.