New efficient and naturally parallelizable time integration algorithm applied to sequential domains for DG-TD

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The discontinuous Galerkin time-domain (DGTD) methods are promising in transient analysis of large and multiscale problems. Based on the idea of domain decomposition, the DG method can handle problems too large to be solved by conventional numerical techniques. Basically, the DG method divides an original problem into several well designed subdomains, i.e., split a large system matrix into several smaller and balanced matrices. Thus, once the spatial discretization is defined, an optimal time integration method is crucial.

In a multiscale DG system, different subdomains usually have different densities of discretization, according to the electrical size of the structures contained in each subdomain. Furthermore, in each subdomain, the smallest discretization length defines the stability criterion for explicit time stepping schemes; hence, a multiscale DG system has a set of hugely different required time steps, and being the smallest of them the global explicit time step. It is clear that an entire multidomain DGTD system solved by a single explicit scheme is extremely inefficient.

Implicit time stepping schemes are unconditionally stable, thus subdomains with electrically small structures can use larger time steps. This advantage has a computational cost in the density of matrices to be inverted, and the complexity of algorithms to solve the DG linear system. Several works have proposed algorithms to take advantage of implicit time stepping without dramatically increasing their computational costs.

This work presents an algorithm that exploits the sequential way in which the subdomains are usually placed in layered structures. The consequence of this connection between subdomains is a coupled linear system with the form of a block tri-diagonal system. This system is transformed into a block LDU (Lower-Diagonal-Upper) decomposition, resulting in a free-of-iteration, very efficient and highly parallelizable algorithm. Numerical experiments show the benefits of this new method for sequential systems, in comparison to algorithms based on iterative solvers or using a different decomposition (e.g., LU).