A Two-Level Nested FETI/FETI-DP Domain Decomposition Method

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The dual primal finite element tearing and interconnecting method (FETI-DP) (Y. J. Li and J. M. Jin, *IEEE Trans. Antennas Propagat.*, vol. 55, no. 10, pp. 2803-2810, 2007) is a stable domain decomposition method when dealing with subdomain conformal meshes for time-harmonic electromagnetic problems. Later, this method is further extended to nonconformal meshes (M. F. Xue and J. M. Jin, *IEEE Trans. Antennas Propagat.*, vol. 60, no. 9, pp. 4291-4305, 2012). To the best of our knowledge, no matter the mesh is conformal or nonconformal, the existing FETI-DP algorithms are limited to the case with a conformal subdomain geometry, which means subdomain interfaces have a one-on-one correspondence. However, in some practical applications, we need to mesh different parts of the entire computational domain (denoted as regions in the following context) separately, and then decompose these separate meshes into subdomains independently. In this case, the connectivity between two subdomains across the region interfaces could be arbitrary; and the existing FETI-DP algorithms are no longer applicable.

In this paper, a two-level nested FETI/FETI-DP method is proposed to bypass such a challenging situation. If we look at the problem in a hierarchical manner, from the coarsest level to the finest level, we have an entire domain, regions, subdomains, and tetrahedral elements. The basic idea of the proposed method is to employ the nonconformal FETI to solve the entire-domain problem by regarding regions as "subdomains," and find out the partial solution in each region using the conformal FETI-DP for the true subdomains in each region. To be more specific, on the region level, we can regard each region as a "subdomain" and solve the entire domain using one-level FETI. As usual, we use an iterative solver to solve the global interface problem first, and then recover the solution in each region by using the solved boundary condition. In each iterative step, we need to solve each region once for a given right-hand-side (in the one-level FETI scheme, this means one-time forward-backward substitutions). For this, we use an imbedded lower-level FETI-DP for each region (with conformal meshes). Consequently, we obtain a natural inner-outer loop iteration scheme (X. C. Wang, Z. Peng, K.-H. Lim, and J.-F. Lee, *IEEE Trans. Electromagn. Compat*., vol. 54, no. 2, pp. 375-388, 2012). Note that, at each iterative step, seeking the partial solution of every region can be done in parallel. Furthermore, the conditioning of the global interface problem on the region level should be much better than that on the subdomain level, as it is at a coarser level. Therefore, faster convergence of the outer loop can be achieved, as compared to that of one-level FETI-DP on the same mesh and decomposed directly into subdomain domains. We will demonstrate the efficiency of the proposed nested method for solving three-dimensional vector wave radiation and scattering problems.