Hardy Space Transparent Boundary Conditions Within An Interior Penalty Discontinuous Galerkin Framework

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The numerical representation of asymptotic boundary conditions on Maxwell fields has long been a problem in Computational Electromagnetics. In local PDE-based formulations, the representation of a proper transparent boundary condition that will generate fields that satisfy these conditions is a challenging problem that has a number of solutions, including absorbing boundary conditions, perfectly matched layers (PMLs), boundary integral hybridization, and infinite elements. Each of these methods has attendant advantages and disadvantages, depending upon the type of analysis being performed. Modal analysis remains particularly difficult, as all of these methods, with the exception of PMLs, destroy the structure of what would otherwise be a linear eigenvalue problem on a closed domain. Even so, PMLs require a multi-parameter optimization, and may still produce spurious eigensolutions unrelated to the usual issues of properly conforming discretization spaces.

Recently, Hardy space methods have been demonstrated as a viable alternative to PMLs for modal analysis of open systems within a curl-conforming finite element framework (Nannen et. al., arXiv:1103.2288, 2011). These methods have a single adjustable parameter, and are amenable to a simple analysis that can ascertain whether or not a mode is spurious (B. Kettner, FU-Berlin Dissertation, 2012). The essence of this framework is the observation that the Laplace transform of incoming and outgoing solutions to the Maxwell problem along a generalized radial variable admit holomorphic extensions on different halves of the complex plane. These half planes can be mapped onto the complex unit disk via a Möbius transform, which has a convenient orthonormal basis, namely Fourier modes. This basis is utilized in the discretization of a Laplace-Möbius transformed exterior problem which i.) preserves the linear eigenvalue structure of the problem, ii.) requires a single parameter that describes the slope of the line that partitions the complex plane, and iii.) allows for the selection of either incoming or outgoing modal solutions.

In this work, we will present a formulation of the Hardy space method for an Interior Penalty Discontinuous Galerkin discretization of the Maxwell eigenproblem. Comparisons will be made to the Hardy space framework for standard curl-conforming Finite Element discretizations, as well as PMLs applied to both DG and FE frameworks. Results will be presented that demonstrate efficacy in finding modal solutions for dielectric resonators and leaky wave structures, among others.