On the Numerical Calculation of Integrals Occurring in the Scattering by Circular Objects

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The Method of Moments (MoM) is widely preferred as a full-wave technique for the analysis of the electromagnetic scattering by objects. A major issue when dealing with MoM is related to the time needed to fill in the scattering matrix. The elements of the scattering matrix are usually singular integrals to be numerically calculated, leading to a high computational burden. If the problem is formulated in the spectral domain, the integrands are slowly decaying, possibly oscillating, functions defined on the whole real axis.

Circular objects, such as disk or annular ring, can be analyzed by means of entire domain expansion functions taking into account the correct edge behavior of the unknown (W.C. Chew and J.A. Kong, Radio Sci., 21, 2590-2598, 1980; G. Panariello, F. Schettino and L. Verolino, AP Symp., 818-821, 1999), thus requiring the calculation of integrals of two (for the disk) or four (for the annular ring) Bessel functions. In such a way the number of unknowns, and therefore the dimension of the scattering matrix, can be kept very low even when high accuracy is required. The aim of this contribution is to show how to dramatically reduce the computational time of the integrals involved.

In free space, the integral under investigation is

$$I_{km}^{lp}(a,b)=\int_0^\infty \left[\sqrt{w^2-k_0^2}\right]^l J_m(wa)\,J_k(wa)\,[J_m(wb)\,J_k(wb)]^p\, w^{(1-2l)}\,dw \qquad (1)$$

where k_0 is the wave number, $l=\pm 1$ depending on the polarization, and $p=0$ or $p=1$ for the disk or the annular ring, respectively. A highly efficient way to compute (1) consists in a suitable deformation in the complex plane of the integration path. As a matter of fact, by resorting to the Cauchy's Theorem, it is possible to "move" the integration path to the imaginary axis, thus transforming the Bessel functions from first kind into second kind, exponentially decaying. The details of the procedure will be given at the conference, due to the lack of space. Only the calculation time improvement is given here, summarized in the table below for different values of the parameters in the case of the annular ring.

Accuracy: 10^{-8}		Accuracy: 10^{-10}	
Calculation time [s]		Calculation time [s]	
Unmodified	Modified	Unmodified	Modified
6.02	0.02	43.46	0.07
4.62	0.02	61.47	0.02
5.99	0.02	57.35	0.03
4.65	0.03	44.20	0.03