On using Charge as an Additional Unknown in the EFIE-hd to Improve Mesh Stability

Jin Cheng* and Robert J. Adams
Electrical & Computer Engineering, University of Kentucky
Lexington, KY, 40506-0046

It has been recently observed that a new electric field integral equation (EFIE) based formulation that relies on the Helmholtz decomposition (HD) (EFIE-hd) of the current can overcome the low frequency breakdown problem of the EFIE. It has been demonstrated that the EFIE-hd is frequency stable and provides accurate solutions for the electric and magnetic fields at high and low frequencies. While the resulting EFIE-hd is frequency stable, it is not stable with respect to mesh refinement. The purpose of this work is to obtain an improved formulation that is also stable with mesh refinement by augmenting the original EFIE-hd with the continuity equation (referred to as EFIE-hdc) and including charge as additional set of unknowns with appropriate diagonal scaling.

The EFIE-hd relies on the Helmholtz decomposition (HD) of the current in order to obtain a formulation that is stable at all frequencies. Unlike the other approaches, the HD of the vector basis used for the current is accomplished via appropriate surface integral constraints. This formulation also does not require a HD of the tangential electric field to obtain a stable linear system. For these reasons, the EFIE-hd enables the use of standard locally corrected Nyström (LCN) discretization methods for the resulting formulation. The current is decomposed as rotational current ($\mathbf{J}_R$) and irrotational current ($\mathbf{J}_I$) via the HD. Substituting $\mathbf{J}_R$ and $\mathbf{J}_I$ into the regular EFIE yields an under-constrained system if a standard Nyström basis is used for the currents. Therefore global constraints on the spaces used for $\mathbf{J}_R$ and $\mathbf{J}_I$ are imposed. To overcome the low frequency breakdown, the divergence of the EFIE is also incorporated. Thus with the appropriate scaling of the currents, the resulting over-determined system can be solved by the method of least squares.

By including charge as additional set of unknowns the hyper singular kernel (scalar potential term) acting on $\mathbf{J}_I$ in regular EFIE can be rewritten as a singular kernel acting on the charge. Thus by appropriate diagonal scaling, the EFIE-hdc system matrix is stable with respect to mesh refinement. A similar diagonal scaling strategy was previously applied to the augmented electric field integral equation (AEFIE) with LCN implementation to achieve mesh refinement stability.