A Fast Surface Integral Equation Solver for the Radiative Transfer Equation

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Long-distance propagation of electromagnetic waves in weakly fluctuating random media often is described by radiative transfer equations (RTEs). RTEs find applications in subjects ranging from astrophysics to atmospheric science and remote sensing. Recently RTEs also have been used to model light propagation in diffusive biological tissue with application to optical imaging. Frequently, the vector nature of the waves is neglected, and scalar RTEs are used in lieu of their matrix-valued counterparts.

Analytic solutions to the steady state RTE are limited to very simple sources and media. To use the equation for simulating realistic propagation phenomena, numerical methods are called for. Both matrix-valued and scalar RTEs have probabilistic representations that permit their solution via Monte-Carlo methods (Spanier et al, Monte Carlo principles and neutron transport problems, Reading, MA: Addison-Wesley Pub. Co, 1969; Bal et al., SIAM Appl. Math., pp. 1639-1666, 2000); while easy to implement, these methods converge slowly and hence are computationally expensive. Finite difference and finite element RTE solvers (Jin et al., *Proc. SPIE* 7557, Multimodal Biomedical Imaging V, 75570S, 2010; Tarvainen et al., Appl. Opt., pp. 876-886, 2005), while deterministic in nature, call for a discretization of the radiation intensity in the five-dimensional space spanned by three spatial and two angular variables; not surprisingly, these solvers too are computationally expensive and presently only can be applied to small scale problems.

This paper presents a surface integral equation (SIE) formalism for solving RTEs in piecewise homogeneous media. SIEs pertinent to the solution of transport phenomena have been known for decades, but only solved in planar geometries because of difficulties in computing RTE Green functions (Case, Proc. Of the AMS and SIAM, pp. 17-36, 1969). Here, we use a scheme proposed by Schotland and coworkers to efficiently evaluate RTE Green functions by means of so-called rotated reference frames (Panasyuk et al. *Journal of Physics A: Mathematical and General* p. 115, 2005). We discretize SIEs pertinent to the RTE using spatial pulse basis functions and truncated harmonic angular expansions, and use fast multipole-inspired methods to rapidly evaluate radiation intensities produced by known boundary sources. By reducing the dimensionality of the problem and using fast summation techniques, the proposed SIE solver requires far fewer computational resources than the above-referenced differential equation solvers. We demonstrate the method by solving the 2D scalar RTE for geometries relevant to biomedical imaging, and discuss extensions of the proposed scheme to 3D and matrix-valued equations.