The Duality Relation for 2D Complementary Optical Nanocircuits

Juan D. Baena* [1], Julián D. Ortiz [1], Ricardo Marqués, and Vicente Delgado[2]
(1) Universidad Nacional de Colombia, Bogota, Colombia
(2) Universidad de Sevilla, Seville, Spain

A few years ago, Nader Engheta and co-workers theoretically extended the circuit theory to the optical spectrum by arranging plasmonic and non-plasmonic elements into a nanostructure of total size much smaller than the used wavelength (N. Engheta, Science, vol. 317, p. 1698, 2007). The recipe was simple: $\mathrm{Re}(\varepsilon) > 0$ for nanocapacitors, $\mathrm{Re}(\varepsilon) < 0$ for nanoinductors, $\mathrm{Im}(\varepsilon) > 0$ for nanoresistors, $\varepsilon \to 0$ for insulators, and $\varepsilon \to \infty$ for connectors. On the other hand, in RF and microwaves, it is well known that the duality between two complementary planar circuits allows for getting the electromagnetic behavior of one structure once its complementary is solved (H. G. Booker, *J. Inst. Elect. Eng.*, vol. Pt. III-A, p. 620, 1946). This duality property is summarized by the formula: $Z Z' = \eta_0^2 / 4$, where Z and Z' are the impedances of the original and complementary circuits, reciprocally, and η_0 is the vacuum impedance. However, this duality might not hold in the new framework of optical nanocircuits because metals are not longer good conductors but plasmonic materials. Therefore, duality would be limited just to RF and microwaves.

Now, we have found a similar duality in the frame of 2D optical nanocircuits. Let us focus our attention on Fig. 1 which is showing a single circuit element and its complementary. Green and red regions represent connectors and insulators, respectively, and the white core is supposed to be filled with a certain finite permittivity. When going from the original to the complementary structure, the boundaries are kept equal but the media filling the different regions are changed to new ones with permittivities satisfying everywhere $\varepsilon\varepsilon' = C =$ constant. If the quasi-electrostatic limit is valid (size much smaller than λ) and fields are invariant along the normal to the plane, then we demonstrated that $Z Z' = -1/(\omega^2 C h^2)$, where h is the thickness of the sample. Besides, we demonstrated that this duality relation is still valid for more complex circuits made of many series or parallel connections. We think that these ideas open the door to the design of 2D optical nanocircuits with dual responses.

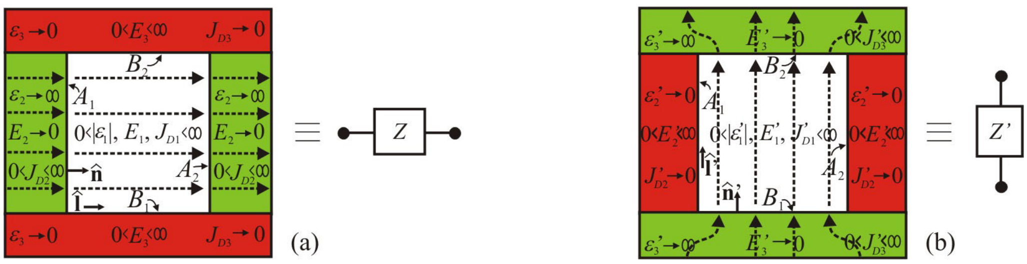

Fig. 1 Original (a) and complementary (b) single circuit elements.