On Use of Inhomogeneous Media For Elimination of Inverse Problem Ill-Posedness

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In our previous work (Okhmatovski, et al., IEEE TAP vol. 60, no. 5, 2012) we demonstrated that the ill-posedness of the inverse problem can be eliminated, if the problem of object reconstruction is staged in the medium exhibiting focusing properties and the scattered field is collected in properly defined locations. The desired focusing properties of the media can be realized using either conventional lenses, mirrors, and antenna arrays or novel materials supporting propagation of the evanescent waves. In this work we utilize Luneburg lens (Henry Jasik, "The Electromagnetic Theory of the Luneburg Lens", 1954) as an example in which focusing properties required for making inverse problem well-posed are realized through inhomogeneity of the media. The lens is responsible in converting the spherically emitted waves of the contrast sources into plane waves. As a result, the waves emitted by different regions of the contrast source are concentrated at different regions of observation. Observing the scattered field at these regions casts the inverse problem into a well-posed form.

We prove the concept numerically by reconstructing material properties of a thin layer conformal to the surface of the Luneburg lens. In the experiment we immersed the object of interest in a background dielectric of relative permittivity of 24. We then take inhomogeneous permittivity of the Luneburg lens appropriately increasing to the value of 48 at its center. In the inverse problem formulation the inhomogeneity of the Luneburg lens and the homogeneous medium of permittivity 24 are treated as the background medium described by Green's function. The latter is easily computable with use of a direct solver (e.g. Richmond, IEEE TAP vol. 13, no. 3, 1965). The focusing properties of the background medium Green's function cast the inverse source problem into a well-posed form allowing for its direct inversion with respect to the unknown distribution of the contrast sources in the thin object conformal to the lens. The permittivity contrast of the object is subsequently obtained from the found contrast source using volumetric equivalence principle.