Detection of Degenerate Eigenmodes in Symmetric Cavities and Their Computation Using Boundary Element Method

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The determination of the eigenfrequencies and the associated eigenmodes of closed structures is a typical task in electromagnetic field computations. Practical examples leading to a mathematical eigenvalue problem are the determination of resonant frequencies of accelerator cavities or the calculation of cut-off frequencies of hollow waveguides. In this context the question according to the existence of degenerate eigenstates arises. Degeneracy means that there exist two or more linearly independent eigenfunctions belonging to the same eigenvalue. This effect is often undesirable and should be avoided during the design phase of such devices so that a theoretical prediction of possible degeneracies could be helpful.

The geometric symmetry of a structure specified by its symmetry group is mainly responsible for the appearance of this phenomenon. Therefore, group theoretical methods are the appropriate mathematical tool for its study. It is well-known that the eigenspaces of an arbitrary resonator are representation spaces of the underlying symmetry group. Each eigenspace can be expanded in the irreducible representations of this group. Hence, the knowledge of the symmetry group of a physical system allows the identification of degeneracies and enables a classification of the eigenmodes with respect to symmetry (e.g. S. Sakanaka, Phys. Rev. ST Accel. Beams, 2005).

This presentation consists of two parts. In the first part the relation between a geometric symmetry of a cavity resonator and the minimum degree of degeneracy will be pointed out and illustrated by means of a rectangular waveguide. Furthermore, the splitting of degenerate eigenstates of a cubic cavity caused by a small perturbation of the geometric symmetry will be discussed. The second part focuses on the numerical analysis of homogeneous cavities using the Boundary Element Method. In addition, the Finite Group Method based on the harmonic analysis on finite groups (R.P. Tarasov, Comp. Math. and Math. Phys., 32, 9, 1992) is applied to transform the surface integral equation into a set of N independent equations each defined merely on a symmetry cell. The reduced structure is given by just 1/N of the part of the whole domain, where N represents the order of the underlying symmetry group. Thus, the computational effort decreases by a factor depending on the symmetry group. Finally, the discretization of the transformed integral equation leads to a non-linear eigenvalue problem for each symmetry type which can be solved numerically.