Uncertainty and Sensitivity Analysis for Time-Domain Electromagnetics using Polynomial Chaos

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Quantifying the uncertainty in computational electromagnetic models is an essential part of the design and validation process. Uncertainty in the input parameters, such as the dielectric properties and physical dimensions, will 'propagate' through the computational model to introduce uncertainty in the results. Estimates of the uncertainty in the modelled response can be used to set realistic design margins. Similarly, estimates of the sensitivity to the input parameters can indicate which inputs should be targeted to have the greatest reduction in the variability of the response. The Monte-Carlo method is widely used to estimate the uncertainty and sensitivity in numerical models, however, the slow rate of convergence tends to limit its application for computationally large problems, such as the FDTD analysis of microwave circuits or indoor propagation. Recently, methods based on generalized polynomial chaos have been proposed to more efficiently quantify large-scale uncertainty in numerical models. The polynomial chaos method approximates quantities in a stochastic process as the finite summation of orthogonal basis polynomials in the random input parameter space (Xiu and Karniadakis, SIAM J. Sci. Comput., 619–644, 2002). This approach can significantly reduce the computational costs required to estimate the variation or sensitivity in the output parameters, compared to a 'traditional' Monte-Carlo analysis. Uncertainty can be incorporated into the FDTD method by expanding the time-domain electric and magnetic fields using polynomial chaos; previous work in this area has largely focused on uncertainty in the material properties (Edwards *et. al.*, IEEE Trans., EMC-52, 155–163, 2010). The objective of this research is to incorporate uncertainty and variability in the physical dimensions of the problem by locally distorting the FDTD computational mesh. Preliminary results for a microstrip low-pass filter with uncertain stub lengths (due to manufacturing tolerances) show the statistics of the solution agree closely with Monte Carlo simulations and are achieved at significantly lower computational cost. Other orthogonal basis functions, such as wavelets, could also be applied to more efficiently span the random parameter space and provide multi-resolution analysis.

Various techniques have been developed to estimate the local and global sensitivities in computational electromagnetic problems, including perturbation, finite-differences and the adjoint variable method (AVM). However, in many cases the results are only valid for small changes in the input parameters. Global sensitivities can also be extracted from the polynomial chaos expansion via the Sobol decomposition. As these are typically valid over a larger range, a further goal of this research is provide objective comparisons between the Sobol-sensitivities and those computed via the AVM and other methods.