A Discontinuous Galerkin Time Domain Framework for Periodic Problems

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Numerical methods for solving the Maxwell Equations subject to Bloch-Floquet periodic boundary conditions have a number of important applications, ranging from conventional microwave applications such as FSS, to 'metamaterials' that effect a novel electromagnetic response using similar physics. Time domain analysis of these structures is particularly useful for studying the effects of nonlinear loading or under interrogation by broadband signals. In local PDE-based formulations of the periodic Maxwell problem the proper satisfaction of boundary conditions is challenging as naive formulations lead to non-causal constraints on the fields under oblique incidence. Fortunately, a simple linear transformation of the fields lead to a re-formulation of the problem that remains causal at arbitrary incidence (L.E.R. Petersson and J.M. Jin, IEEE TAP, 54(1), 2006). While standard Finite Element discretizations have been applied to this formalism, to the best of our knowledge Discontinuous Galerkin (DG) methods have not.

DG has a number of salient features worth exploiting, particularly local time stepping methods for multiscale geometries, and amenability to non-conformal meshing (S. Dosopolous, B. Zhao, and J.F. Lee, J. Comp. Phys., in press, 2012). The latter, in particular, is useful for periodic systems in which it may not always be simple to generate a mesh that is face-conformal across periodically matched boundaries. In this contribution, we will present a DG framework for the transformed Bloch-Floquet periodic time domain Maxwell equations with a focus on implementing numerical fluxes that satisfy an appropriate Rankine-Hugoniot condition in the transformed variables. Results will validate our formulation, and demonstrate utility for studying both nonlinearly loaded systems and structures in which Floquet modes beyond zeroth order are excited. Viable methods for implementing transparent boundary conditions for the latter will be discussed as well.