Analysis of Nonlinear Electromagnetic Problems Using Time-Domain Finite Element Method

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Many materials in the nature or fabricated in laboratories exhibit nonlinear behavior where the permittivity and/or permeability is a function of the magnitude of the electric or magnetic field intensity. Such nonlinearity results in a nonlinear system of Maxwell's equations, which are more challenging to solve. Generally speaking, there are two kinds of methods commonly adopted to solve nonlinear problems. The first kind of methods solves the linearized versions of the original problems, while the second kind of methods deals with the nonlinear problems directly. Although more difficult to solve with a high efficiency, the second kind of methods is more desirable in order to capture the nonlinear phenomenon more accurately.

Among various nonlinear solvers developed in the applied mathematics community, the Newton-Raphson method is widely used due to its second-order convergence, if started from an initial guess sufficiently close to the true solution. However, in a real application, there is no straightforward way to choose a good initial guess. As a result, the Newton-Raphson method could be unstable or divergent in solving complicated problems. Another difficulty of applying the Newton-Raphson method is that the Jacobian matrix needs to be updated and solved in every Newton iteration, which could be very time consuming for solving large problems.

When the nonlinear electromagnetic problems are formulated and solved with the finite element method in time domain (J. M. Jin, *The Finite Element Method in Electromagnetics*, *Second Edition*. New York: Wiley, 2002), the numerical solution at a previous time step can be used as a natural choice for the initial guess of the Newton iteration at the successive time step. The convergent of the Newton-Raphson method can be achieved if the time step is small enough such that the solutions of two successive time steps are sufficiently close. In this presentation, we employ the time-domain finite element method (TDFEM) combined with the Newton-Raphson method and its variations to solve nonlinear electromagnetic problems. We first study the TDFEM formulation of nonlinear problems, and then show how it can be solved with Newton(-like) methods. We demonstrate and compare the performance of different nonlinear solvers including the basic Newton-Raphson method and its variations through a number of numerical examples.