Singular Hierarchical Vector Elements of the Additive Kind for Triangular Meshes

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New hierarchical vector elements of arbitrary polynomial order were recently derived in **[(1)** R. D. Graglia, A.F. Peterson, F.P. Andriulli, "Curl-conforming hierarchical vector bases for triangles and tetrahedra," *IEEE TAP*, vol. 59, no. 3, pp. 950-959, 2011; **(2)** R.D. Graglia, A.F. Peterson, "Hierarchical curl–conforming Nédélec elements for quadrilateral and brick cells," *IEEE TAP*, vol. 59, no. 8, pp. 2766-2773, 2011; **(3)** R.D. Graglia, A.F. Peterson, "Hierarchical curl–conforming Nédélec elements for triangular-prism cells," *IEEE TAP*, vol. 60, no. 7, pp. 3314-3324, 2012; **(4)** R.D. Graglia, A.F. Peterson, "Hierarchical divergence-conforming Nédélec elements for volumetric cells," *IEEE TAP*, vol. 60, no. 11, pp. 5215-5227, 2012**]**. For problems with smooth surfaces or other regular features, these high order hierarchical bases successfully improve accuracy and efficiency. However, for geometries with edges or corners where unbounded fields or other singular types of behavior occur, high degree polynomial expansion functions often do not improve the solution accuracy. Instead, to improve the accuracy of these problems special basis functions are developed that incorporate the singular field behavior. The existing singular basis functions can usually be classified into two types: *substitutive* or *additive* functions. Substitutive basis functions are those for which one or more polynomial basis functions from the original set are removed and replaced by a basis function with an appropriate singular behavior. Additive functions, on the other hand, retain the entire original set and augment it with additional singular basis functions that define the so-called *Meixner* subset. Additive functions are more flexible and can model appropriate field behavior even if the expected singularity is not excited by the source, or if the cells are electrically large. A drawback to the additive approach is that the resulting system of equations is often poorly conditioned, due to the lack of linear independence between the regular and Meixner subspaces except in the immediate vicinity of the singularity. For 2D problems, a *scalar* basis set incorporating the appropriate singularities for the longitudinal field was recently proposed in **[(5)** R.D. Graglia, A.F. Peterson, L. Matekovits, "Singular, hierarchical scalar basis functions for triangular cells," *IEEE TAP*, submitted, Nov. 2012**]**. These singular bases are formed using orthogonal functions to reduce the ill-conditioning of the resulting system matrix. This paper extends the same procedure to derive hierarchical, curl-conforming *singular vector* Meixner subsets on triangular cells. Our new bases have three distinguishing features whose outcomes will be discussed in detail at the Conference: (a) the Meixner vector basis functions are subdivided from the outset into two different groups of edge and face–based functions; (b) in each group, each basis function is obtained from mutually orthogonal scalar functions; (c) the hierarchical vector functions are either symmetric or antisymmetric with respect to the local *parent* variables that describe each cell. The new hierarchical families will be presented at the Conference together with numerical results that compare the performance of these functions to other existing basis families.